Cambridge IGCSE

**Cambridge International Examinations**
Cambridge International General Certificate of Secondary Education

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|---|---|---|---|
| CANDIDATE NAME | | | |
| CENTRE NUMBER | | CANDIDATE NUMBER | |

*0048847567*

**MATHEMATICS** **0580/42**

Paper 4 (Extended) **May/June 2014**

**2 hours 30 minutes**

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator; Geometrical instruments; Tracing paper (optional)

**READ THESE INSTRUCTIONS FIRST**

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For $\pi$, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 1/Level 2 Certificate.



IB14 06_0580_42/2RP


CAMBRIDGE International Examinations

**1** Jane and Kate share \$240 in the ratio $5:7$.

**(a)** Show that Kate receives \$140.

*Answer(a)*

$$\frac{240}{12} \times 7 = 140$$

[2]

**(b)** Jane and Kate each spend \$20.

Find the new ratio Jane's remaining money : Kate's remaining money.
Give your answer in its simplest form.

1 part = 20

$5:7$

$4:6 \rightarrow 2:3$

*Answer(b)* ......2...... : ......3...... [2]

**(c)** Kate invests \$120 for 5 years at 4% per year simple interest.

Calculate the total amount Kate has after 5 years.

$$SI = \frac{P \times n \times I}{100} = \frac{120 \times 5 \times 4}{100} = 24$$

$120 + 24$

*Answer(c)* \$ ......144...... [3]

**(d)** Jane invests \$80 for 3 years at 4% per year compound interest.

Calculate the total amount Jane has after 3 years.
Give your answer correct to the nearest cent.

$4 + 100 = 104$

$80 \times (1.04)^3$

*Answer(d)* \$ ......89.99...... [3]

**(e)** An investment of \$200 for 2 years at 4% per year compound interest is the same as an investment of \$200 for 2 years at $r$% per year simple interest.

Find the value of $r$.

$4 + 100 = 104$

$200 \times (1.04)^2 = 216.32$

Interest $= 16.32$

$$16.32 = \frac{200 \times 2 \times r}{100}$$

$$\frac{16.32 \times 100}{200 \times 2} = r$$

*Answer(e)* $r$ = ......4.08...... [3]

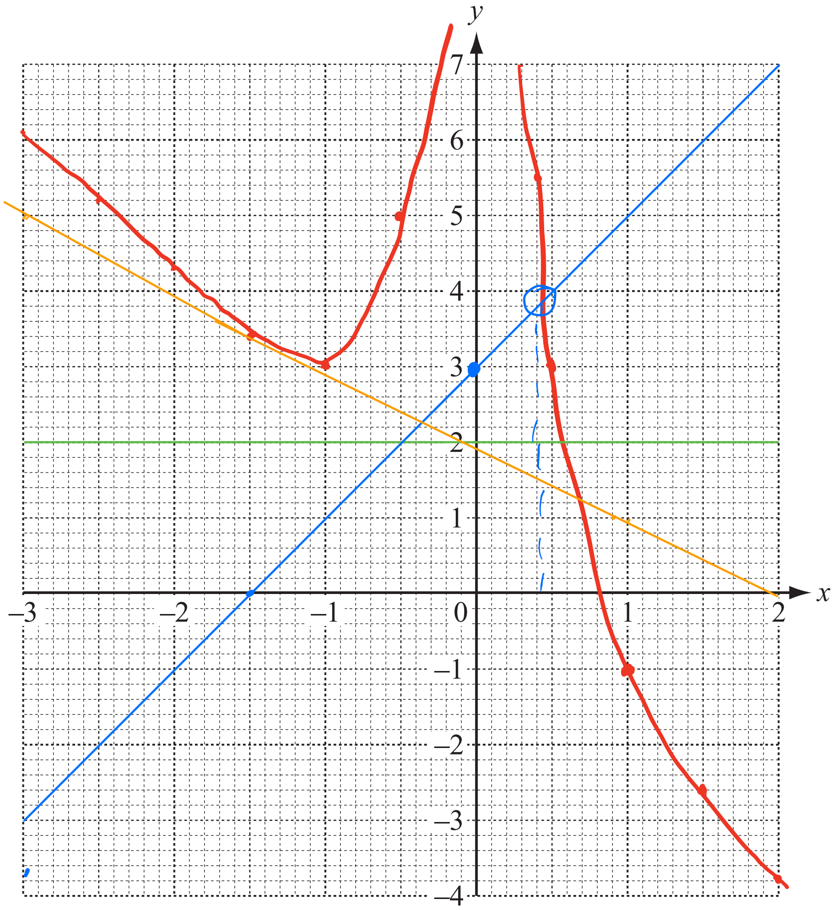
**2** $\quad f(x) = \dfrac{1}{x^2} - 2x, \quad x \neq 0$

**(a)** Complete the table of values for $f(x)$.

| $x$ | –3 | –2.5 | –2 | –1.5 | –1 | –0.5 | 0.4 | 0.5 | 1 | 1.5 | 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | 6.1 | 5.2 | 4.3 | 3.4 | 3 | 5 | 5.5 | 3 | –1 | –2.6 | –3.8 |

[3]

**(b)** On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.4 \leqslant x \leqslant 2$.




[5]

**(c)** Solve the equation $f(x) = 2$.

*Answer(c)* $x =$ 0.55 [1]

**(d)** Solve the equation $f(x) = 2x + 3$.

(0,3) (–1.5,0)

$x = 0 \quad y = 3$

$y = 0 \quad x = -3/2$

*Answer(d)* $x =$ 0.4 [3]

**(e)** **(i)** Draw the tangent to the graph of $y = f(x)$ at the point where $x = -1.5$. done in orange [1]

**(ii)** Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$.

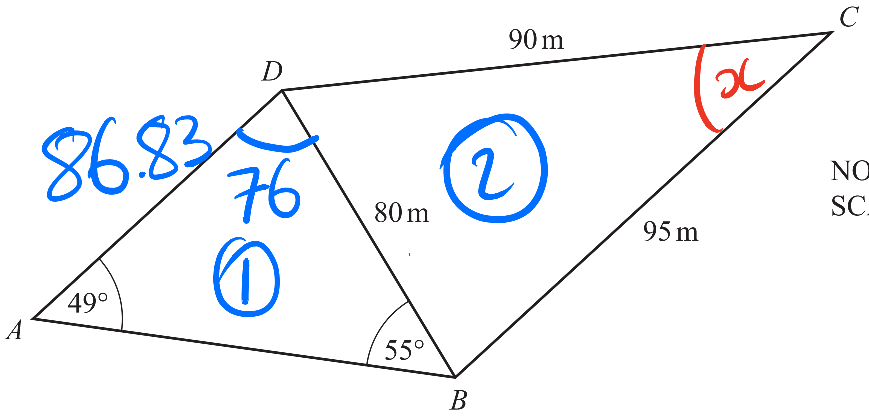
(–3,5)

(–1.5,3.4)

$m = \dfrac{y_1 - y_2}{x_1 - x_2} \quad \dfrac{5 - 3.4}{-3 + 1.5}$

*Answer(e)(ii)* $-16/15$ [2]

3




NOT TO SCALE

The diagram shows a quadrilateral *ABCD*.
Angle *BAD* = 49° and angle *ABD* = 55°.
*BD* = 80 m, *BC* = 95 m and *CD* = 90 m.

**(a)** Use the sine rule to calculate the length of *AD*.

$$\frac{x}{\sin(55)} = \frac{80}{\sin(49)}$$

$$x = \frac{80}{\sin(49)} \times \sin(55)$$

*Answer(a) AD* = 86.83 m [3]

**(b)** Use the cosine rule to calculate angle *BCD*.

$$\cos A = \frac{(95)^2 + (90)^2 - (80)^2}{2(95)(90)}$$

$$A = \cos^{-1}\left(\frac{95^2 + 90^2 - 80^2}{2(95)(90)}\right)$$

*Answer(b)* Angle *BCD* = 51.2 [4]

**(c)** Calculate the area of the quadrilateral *ABCD*.

① $\frac{1}{2} \times 80 \times 86.83 \times \sin(76)$

$+$

② $\frac{1}{2} \times 95 \times 90 \times \sin(51.2)$

*Answer(c)* ......6701...... $m^2$ [3]

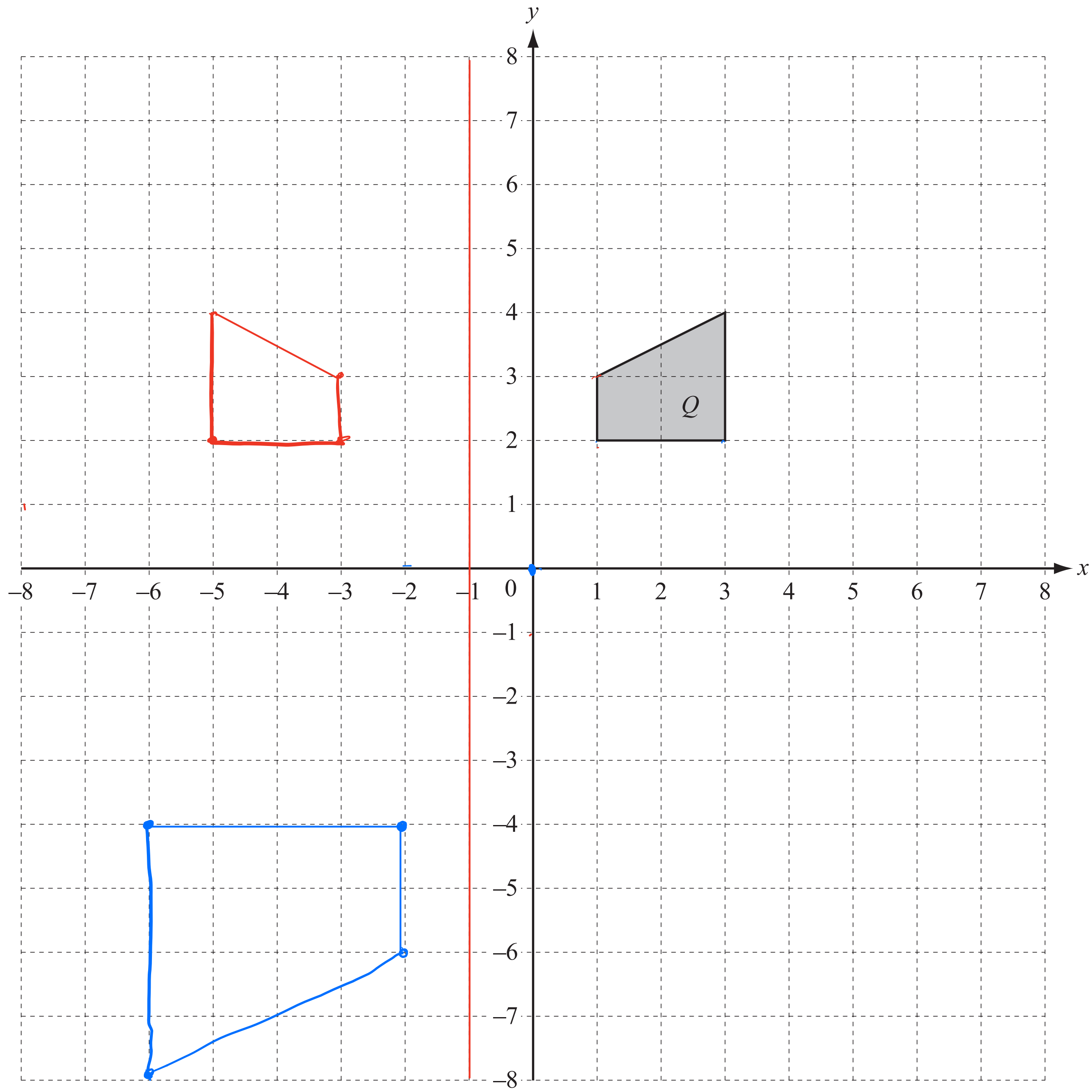
**(d)** The quadrilateral represents a field.
Corn seeds are sown across the whole field at a cost of $3250 per hectare.

Calculate the cost of the corn seeds used.
1 hectare = 10 000 $m^2$

$\frac{6701}{10,000} \times 3250$

*Answer(d)* $ ......2177.83...... [3]

**4**




**(a)** Draw the reflection of shape $Q$ in the line $x = -1$. [2]

**(b)** **(i)** Draw the enlargement of shape $Q$, centre (0, 0), scale factor $-2$. [2]

**(ii)** Find the 2 × 2 matrix that represents an enlargement, centre (0, 0), scale factor $-2$.

$$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$$

*Answer(b)*(ii) $\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$ [2]

**(c)** **(i)** Draw the stretch of shape $Q$, factor 2, $x$-axis invariant. [2]

**(ii)** Find the 2 × 2 matrix that represents a stretch, factor 2, $x$-axis invariant.

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

*Answer(c)*(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

**(iii)** Find the inverse of the matrix in **part (c)(ii)**.

$$\frac{1}{2}\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \text{ or } \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

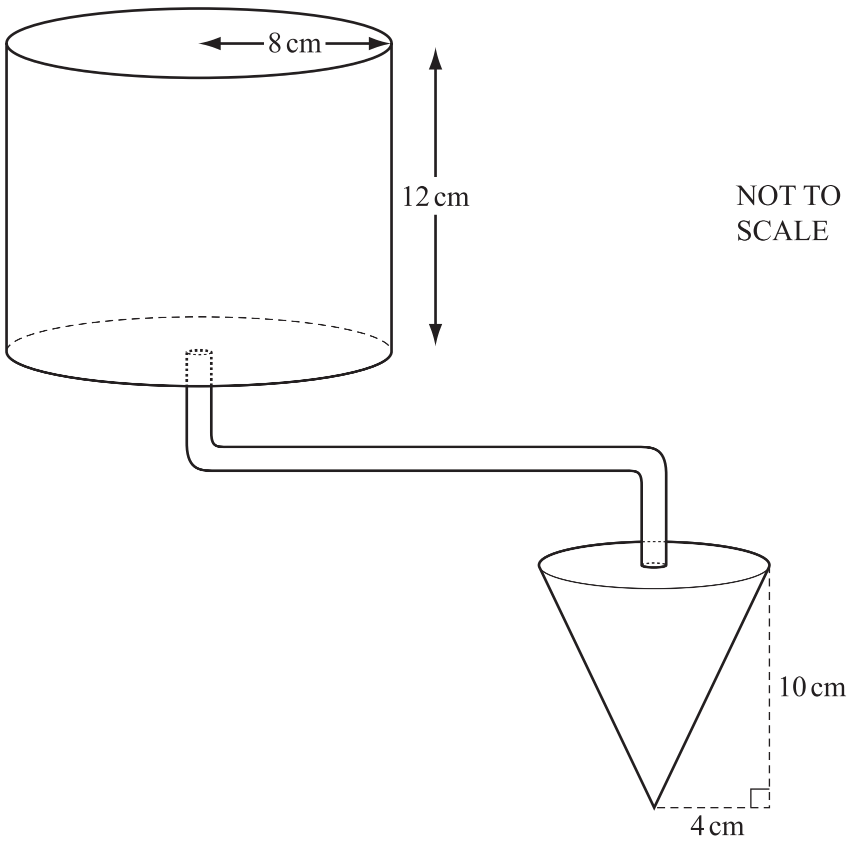
*Answer(c)*(iii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

**(iv)** Describe fully the **single** transformation represented by the matrix in **part (c)(iii)**.

*Answer(c)*(iv) Stretch, scale factor ½ and

the invariant line on the x-axis. [3]

5




NOT TO SCALE

The diagram shows a cylinder with radius 8 cm and height 12 cm which is full of water.
A pipe connects the cylinder to a cone.
The cone has radius 4 cm and height 10 cm.

**(a)** **(i)** Calculate the volume of water in the cylinder.
Show that it rounds to 2410 cm$^3$ correct to 3 significant figures.

*Answer(a)*(i)

$V = \pi r^2 h = \pi(8)^2(12) = 2412.743158$

To 3 significant figures

2410cm$^3$

[2]

**(ii)** Change 2410 cm$^3$ into litres.

*Answer(a)*(ii) 2.41 litres [1]

$2410 \div 1000$

2.41 litres

**(b)** Water flows from the cylinder along the pipe into the cone at a rate of 2 cm$^3$ per second.

Calculate the time taken to fill the empty cone.
Give your answer in minutes and seconds correct to the nearest second.

[The volume, $V$, of a cone with radius $r$ and height $h$ is $V = \frac{1}{3}\pi r^2 h$.]

Volume of Cone $= \frac{1}{3} \times \pi \times 4^2 \times 10 = \frac{160}{3}\pi$

$\frac{160}{3}\pi \div 2 = 83.775804$ is the total time in Seconds

1 minute = 60 seconds

*Answer(b)* ......1...... min ......24...... s [4]

**(c)** Find the number of empty cones which can be filled completely from the full cylinder.

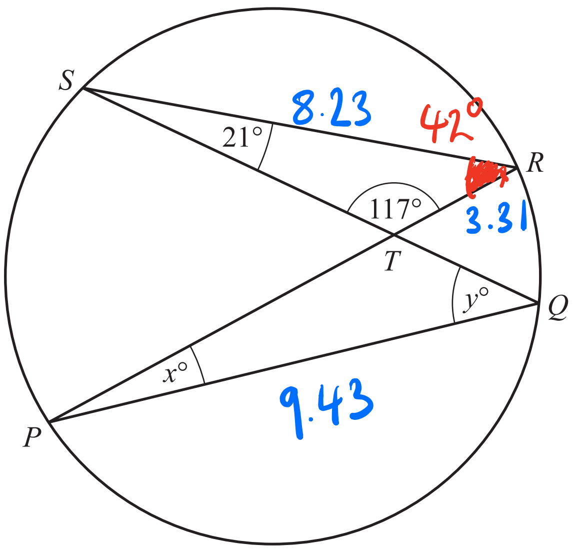
$$\frac{2410}{\frac{160}{3}\pi} = 14.4$$

Volume of cone $= \frac{1}{3} \times \pi \times 4^2 \times 10$

Only 14 will be Full

*Answer(c)* ......14...... [3]

**6**




NOT TO SCALE

**(a)** The chords $PR$ and $SQ$ of the circle intersect at $T$.
Angle $RST = 21°$ and angle $STR = 117°$.

**(i)** Find the values of $x$ and $y$.

*Answer(a)*(i) $x$ = ......21......

$y$ = ......42...... [2]

**(ii)** $SR = 8.23$ cm, $RT = 3.31$ cm and $PQ = 9.43$ cm.

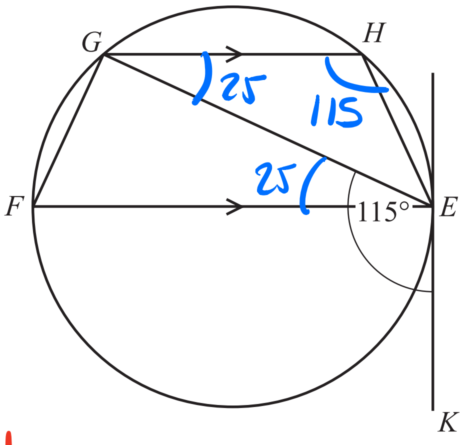
Calculate the length of $TQ$.

$$\frac{3.31}{TQ} = \frac{8.23}{9.43}$$

$$TQ = \frac{3.31 \times 9.43}{8.23}$$

*Answer(a)*(ii) $TQ$ = ......3.79...... cm [2]

**(b)** *EFGH* is a cyclic quadrilateral.
*EF* is a diameter of the circle.
*KE* is the tangent to the circle at *E*.
*GH* is parallel to *FE* and angle *KEG* = 115°.




NOT TO SCALE

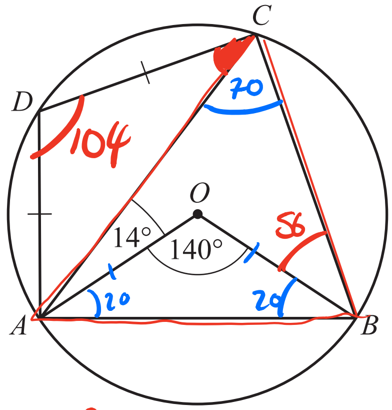
FEG = 25 as 115 − 90
(Tangents meet center at 90°)

Calculate angle *GEH*.

EGH = 25 as it is an alternate angles
EHG = 115 by alternate segment theorem
GEH = 180 − 115 − 25

*Answer(b)* Angle *GEH* = 40 ............................................ [4]

**(c)** *A*, *B*, *C* and *D* are points on the circle centre *O*.
Angle *AOB* = 140° and angle *OAC* = 14°.
*AD* = *DC*.




NOT TO SCALE

180 − 140 = 40

$\frac{40}{2} = 20$

Triangle ABC needs to be 180°

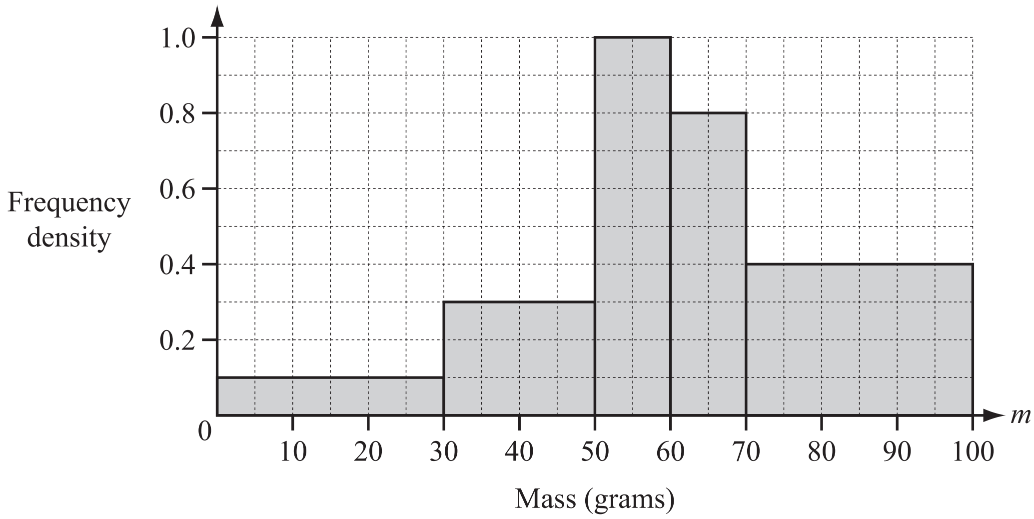
Calculate angle *ACD*.

so
OBC = 56
As ABCD is a cyclic quadrilateral
ADC = 180 − (20+56) = 104
As ADC is isosceles Triangle

*Answer(c)* Angle *ACD* = 38 ............................................ [5]

Angle ACD = $\frac{180-104}{2}$

**7 (a)**




The histogram shows some information about the masses ($m$ grams) of 39 apples.

**(i)** Show that there are 12 apples in the interval $70 < m \leqslant 100$ .

*Answer(a)*(i) $30 \times 0.4 = 12$

[1]

**(ii)** Calculate an estimate of the mean mass of the 39 apples.

| | 0 to 30 | 30 to 50 | 50 to 60 | 60 to 70 | 70 to 100 |
|---|---|---|---|---|---|
| $x$ | 15 | 40 | 55 | 65 | 85 |
| F | 3 | 6 | 10 | 8 | 12 |
| F$x$ | 45 | 240 | 550 | 520 | 1020 |

$$\text{mean} = \frac{\text{total F}x}{\text{total F}} = \frac{2375}{39}$$

*Answer(a)*(ii) 60.9 g [5]

**(b)** The mean mass of 20 oranges is 70 g.
One orange is eaten.
The mean mass of the remaining oranges is 70.5 g.

Find the mass of the orange that was eaten.

$$\frac{x}{19} = 70.5$$

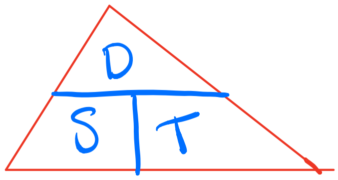
$x = 70.5 \times 19$

$x = 1339.5$

$\frac{y}{20} = 70 \quad y = 70 \times 20 = 1400$

$1400 - 1339.5$

*Answer(b)* 60.5 g [3]




**8** The distance a train travels on a journey is 600 km.

**(a)** Write down an expression, in terms of $x$, for the average speed of the train when

**(i)** the journey takes $x$ hours,

$S = \frac{D}{T}$ $\quad S = \frac{600}{x}$

*Answer(a)*(i) $\frac{600}{x}$ km/h [1]

**(ii)** the journey takes $(x + 1)$ hours.

*Answer(a)*(ii) $\frac{600}{x+1}$ km/h [1]

**(b)** The difference between the average speeds in **part(a)(i)** and **part(a)(ii)** is 20 km/h.

**(i)** Show that $x^2 + x - 30 = 0$.

Longer time = slower speed

*Answer(b)*(i)

$$\frac{600}{x} - \frac{600}{x+1} = 20$$

Cross multiply

$$600(x+1) - 600(x) = 20(x)(x+1)$$

$$600x + 600 - 600x = 20(x^2 + x)$$

$$600 = 20x^2 + 20x$$

$$20x^2 + 20x - 600 = 0$$

$\div 20$

$$x^2 + x - 30 = 0$$

[3]

**(ii)** Find the average speed of the train for the journey in **part(a)(ii)**.
Show all your working.

$$x^2 + x - 30 = 0$$

$$(x+6)(x-5) = 0$$

$$x = -6 \text{ or } x = 5$$

x can't be negative as it will give a negative speed

$$\text{Speed} = \frac{600}{x+1}$$

$$\text{Speed} = \frac{600}{5+1} = \frac{600}{6}$$

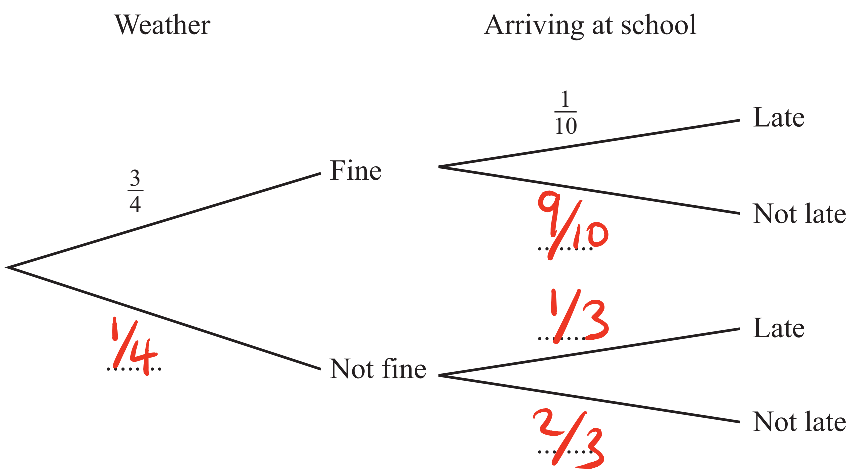
*Answer(b)*(ii) 100 km/h [4]

**9** If the weather is fine the probability that Carlos is late arriving at school is $\frac{1}{10}$.

If the weather is not fine the probability that he is late arriving at school is $\frac{1}{3}$.

The probability that the weather is fine on any day is $\frac{3}{4}$.

**(a)** Complete the tree diagram to show this information.




[3]

**(b)** In a school term of 60 days, find the number of days the weather is expected to be fine.

$\frac{3}{4} \times 60$

*Answer(b)* ........45........ [1]

**(c)** Find the probability that the weather is fine and Carlos is late arriving at school.

$\frac{3}{4} \times \frac{1}{10}$

*Answer(c)* ........$\frac{3}{40}$........ [2]

**(d)** Find the probability that Carlos is not late arriving at school.

$\left(\frac{3}{4} \times \frac{9}{10}\right) + \left(\frac{1}{4} \times \frac{2}{3}\right)$

*Answer(d)* ........$\frac{101}{120}$........ [3]

**(e)** Find the probability that the weather is not fine on at least one day in a school week of 5 days.

1 − Pr(Fine) For 5days

$1 - \left(\frac{3}{4}\right)^5 =$

*Answer(e)* ........781/1024........ [2]

**10** $\mathrm{f}(x) = \frac{1}{x}$, $x \neq 0$ $\mathrm{g}(x) = 1 - x$ $\mathrm{h}(x) = x^2 + 1$

**(a)** Find $\mathrm{fg}\left(\frac{1}{2}\right)$.

$g\left(\frac{1}{2}\right) = 1 - \frac{1}{2} = \frac{1}{2}$

$fg\left(\frac{1}{2}\right) = \frac{1}{\frac{1}{2}} = 2$

*Answer(a)* ........2........ [2]

**(b)** Find $\mathrm{g}^{-1}(x)$, the inverse of $\mathrm{g}(x)$.

$y = 1 - x$ $x = 1 - y$ $y = 1 - x$ $g^{-1}(x) = 1 - x$

*Answer(b)* $\mathrm{g}^{-1}(x) =$ ........$1 - x$........ [1]

**(c)** Find $\mathrm{hg}(x)$, giving your answer in its simplest form.

$g(x) = 1 - x$

$h(g(x)) = (1 - x)^2 + 1$

$(1 - x)(1 - x) + 1$

$1 - 2x + x^2 + 1$

*Answer(c)* $\mathrm{hg}(x) =$ ........$x^2 - 2x + 2$........ [3]

**(d)** Find the value of $x$ when $\mathrm{g}(x) = 7$ .

$1 - x = 7$

$1 - 7 = x$

*Answer(d)* $x =$ ........$-6$........ [1]

**(e)** Solve the equation $\mathrm{h}(x) = 3x$.
Show your working and give your answers correct to 2 decimal places. means Quadratic equation.

$x^2 + 1 = 3x$

$x^2 - 3x + 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{3 \pm \sqrt{9 - (4)(1)(1)}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

*Answer(e)* $x =$ ........2.62........ or $x =$ ........0.38........ [4]

**(f)** A function $\mathrm{k}(x)$ is its own inverse when $\mathrm{k}^{-1}(x) = \mathrm{k}(x)$.

For which of the functions $\mathrm{f}(x)$, $\mathrm{g}(x)$ and $\mathrm{h}(x)$ is this true?

*Answer(f)* ........g(x) and f(x)........ [1]

**Question 11 is printed on the next page.**

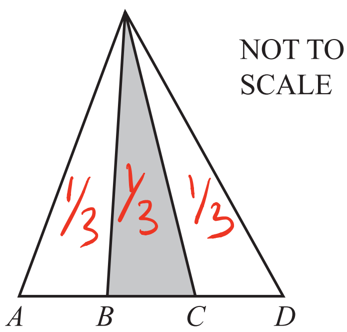
**11** The total area of each of the following shapes is $X$.
The area of the shaded part of each shape is $kX$.

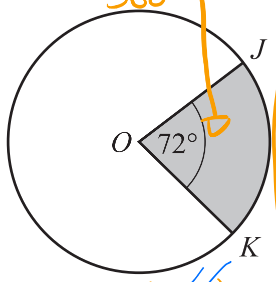
For each shape, find the value of $k$ and write your answer below each diagram.

$\frac{72}{360} \times \pi \times r^2$

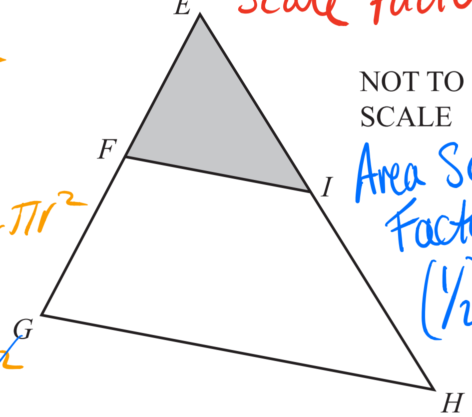
$X = \pi r^2$

Scale Factor $\frac{1}{2}$




$Kx = \frac{72}{360} \times \pi r^2$




Area Scale Factor $(\frac{1}{2})^2 = \frac{1}{4}$

$K\cancel{X} = \frac{1}{3}\cancel{X}$

*AB* = *BC* = *CD*

$k$ = ...$\frac{1}{3}$...

$K(\cancel{\pi r^2}) = \frac{72}{360} \times \cancel{\pi r^2}$

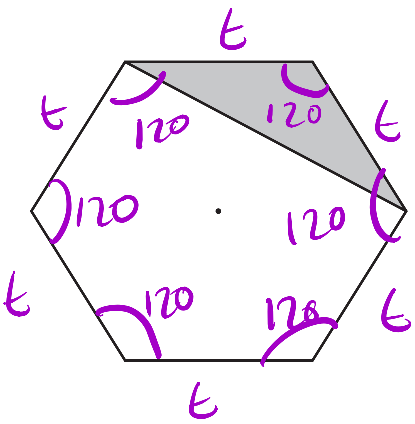
Angle *JOK* = 72°

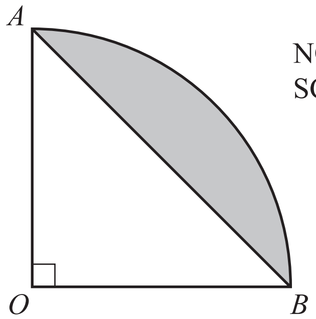
$k$ = ...$\frac{72}{360}$...

*EF* = *FG* and *EI* = *IH*

$Kx = \frac{1}{4}x$

$k$ = ...$\frac{1}{4}$...




See Below

The shape is a regular hexagon.

$(n-2) \times 180 = 4 \times 180 = 720$

$\frac{720}{6} = 120$

$X = 6(\frac{1}{2} \times t^2 \times \sin(120))$

$k$ = ...$\frac{1}{6}$...

The diagram shows a sector of a circle centre $O$.
Angle *AOB* = 90°

$Kx = 6(\frac{1}{2} \times t^2 \times \sin(120))$

$Kx = \frac{1}{6}x$

$K = \frac{1}{6}$

$k$ = ...$1 - \frac{2}{\pi}$...

[10]

$$\text{Sector} = \frac{90}{360} \times \pi r^2 = X.$$

$$\text{Shaded} = \frac{90}{360} \times \pi r^2 - \frac{1}{2} r^2 = KX.$$

$$Kx = \frac{90}{360} \pi r^2 - \frac{1}{2} r^2.$$

$$\frac{90}{360} = \frac{1}{4}.$$

$$K\left(\frac{1}{4} \pi r^2\right) = \frac{1}{4} \pi r^2 - \frac{1}{2} r^2.$$

$$K = \frac{1/4 \pi r^2 - \frac{1}{2} r^2.}{1/4 \pi r^2.} = 1 - \frac{1/2 \cancel{r^2}}{1/4 \pi \cancel{r^2}}$$

$$K = 1 - \frac{1/2}{1/4 \pi} = 1 - \frac{2}{\pi}$$