Cambridge IGCSE

Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME											
CENTRE NUMBER							CANDIDATE NUMBER				

4490987798

MATHEMATICS **0580/42**

Paper 4 (Extended) **May/June 2016**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator Geometrical instruments
Tracing paper (optional)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

1 Mr Chan flies from London to Los Angeles, a distance of 8800 km.
The flight takes 11 hours and 10 minutes.

(a) **(i)** His plane leaves London at 09 35 local time.
The local time in Los Angeles is 8 hours behind the time in London.

Calculate the local time when the plane arrives in Los Angeles.

12:45 .... [2]

(ii) Work out the average speed of the plane in km/h.

$\frac{D}{S \times T}$

$\text{Speed} = \frac{\text{distance}}{\text{Time}} = \frac{8800}{11\frac{1}{6}}$

$\frac{10}{60} = \frac{1}{6}$

788.06 .... km/h [2]

(b) There are three types of tickets, economy, business and first class.
The price of these tickets is in the ratio economy : business : first class = 2 : 5 : 9.

(i) The price of a business ticket is $2350.

Calculate the price of a first class ticket.

$\div 5 \left(\begin{array}{l} 5 \text{ parts} = 2350 \\ 1 \text{ part} = 470 \end{array} \right) \div 5$ $\times 9 \left(\begin{array}{l} 1 \text{ part} = 470 \\ 9 \text{ parts} = 4230 \end{array} \right) \times 9$

$ 4230 .... [2]

(ii) Work out the price of an economy ticket as a percentage of the price of a first class ticket.

$\text{economy} = 2 \times 470 = 940$

$\text{Percentage} = \frac{940}{4230} \times 100 = 22.2\%$

22.2 .... % [1]

(c) The price of a business ticket for the same journey with another airline is $2240.

(i) The price of a first class ticket is 70% more than a business ticket.

Calculate the price of this first class ticket.

$\div 100 \left(\begin{array}{l} 100\% = 2240 \\ 1\% = 22.4 \end{array} \right) \div 100$

$\times 170 \left(\begin{array}{l} 1\% = 22.4 \\ 170\% = 3808 \end{array} \right) \times 170$

$ 3808 .... [2]

(ii) The price of a business ticket is 180% **more** than an economy ticket.

$\times 100$ $1\% = 8$ $\times 100$
$100\% = 800$

Calculate the price of this economy ticket.

$280\% = 2240$ $\div 280$
$\div 280$ $1\% = 8$

$ 800 [3]

(d) Mr Chan hires a car in Los Angeles.
The charges are shown below.

<u>Car Hire</u>

$28.00 per day plus $6.50 per day insurance.

$1.25 for every kilometre travelled after the first 800 km.
The first 800 km are included in the price.

Mr Chan hired the car for 12 days and paid $826.50 .

(i) Find the number of kilometres Mr Chan travelled in this car.

$28 + 6.50 = 34.2$
$12 \times 34.2 = 414$
$826.50 - 414 = 412.5$
$\frac{412.5}{1.25} = 330$

$800 + 330$

1130 km [4]

(ii) The car used fuel at an average rate of 1 litre for every 10 km travelled.
Fuel costs $1.30 per litre.

Calculate the cost of the fuel used by the car during the 12 days.

$\frac{1130}{10} = 113$ Liters

$113 \times 1.30 = 146.9$

$ 146.90 [2]

2 **(a)** Work out the value of x in each of the following.

(i) $3^x = 243$

$3^3 = 27$ $3^4 = 81$ $3^5 = 243$

$x =$ 5 [1]

(ii) $16^x = 4$

$16^{1/2} = 4$

$x =$ $1/2$ [1]

(iii) $8^x = 32$

$8^{5/3} = 32$

$x =$ $5/3$ [2]

(iv) $27^x = \frac{1}{9}$

$27^{-\frac{2}{3}} = \frac{1}{9}$

$x =$ $-2/3$ [2]

(b) Solve by factorisation.

$$y^2 - 7y - 30 = 0$$

Show your working.

Product = -30

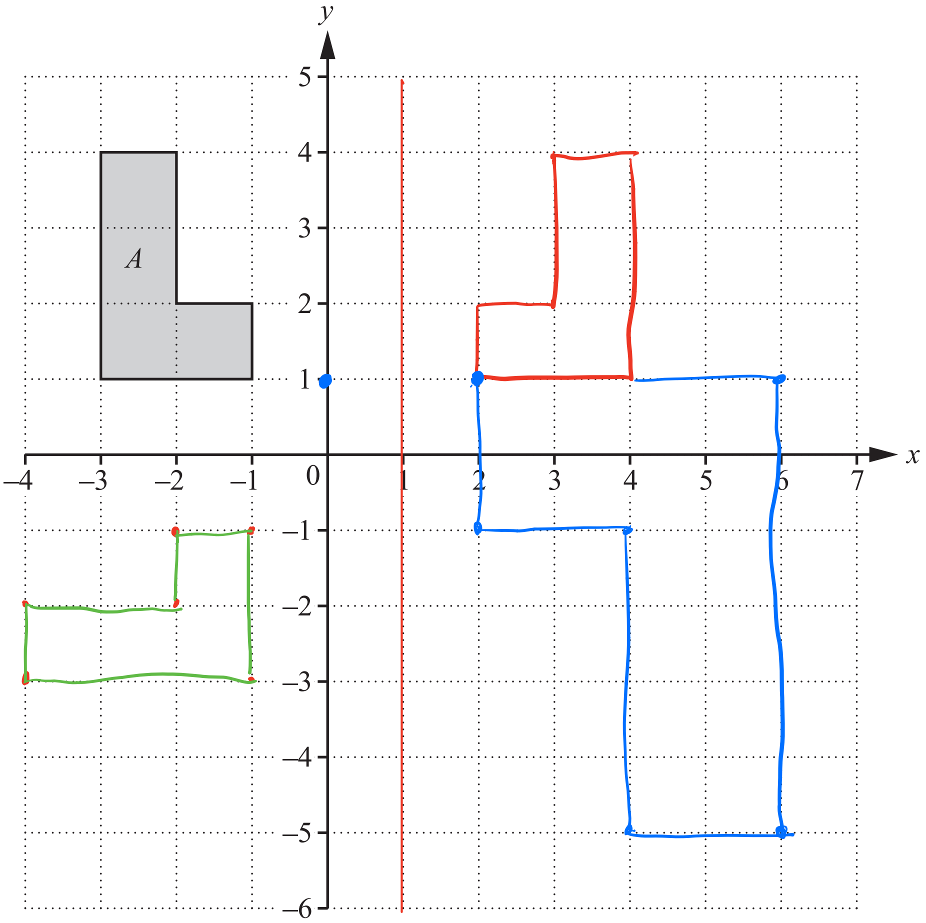
Sum = -7

$(y-10)(y+3)=0$

$y=10$ or $y=-3$

$y =$ 10 or $y =$ -3 [3]

3 **(a)**



On the grid, draw the image of

(i) shape A after a reflection in the line $x = 1$, [2]

(ii) shape A after an enlargement with scale factor –2, centre (0, 1), [2]

(iii) shape A after the transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$. [3]

Rotation 90° Anticlockwise.

(b) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$.

Enlargement, Scale factor 3 at center (0,0)

[3]

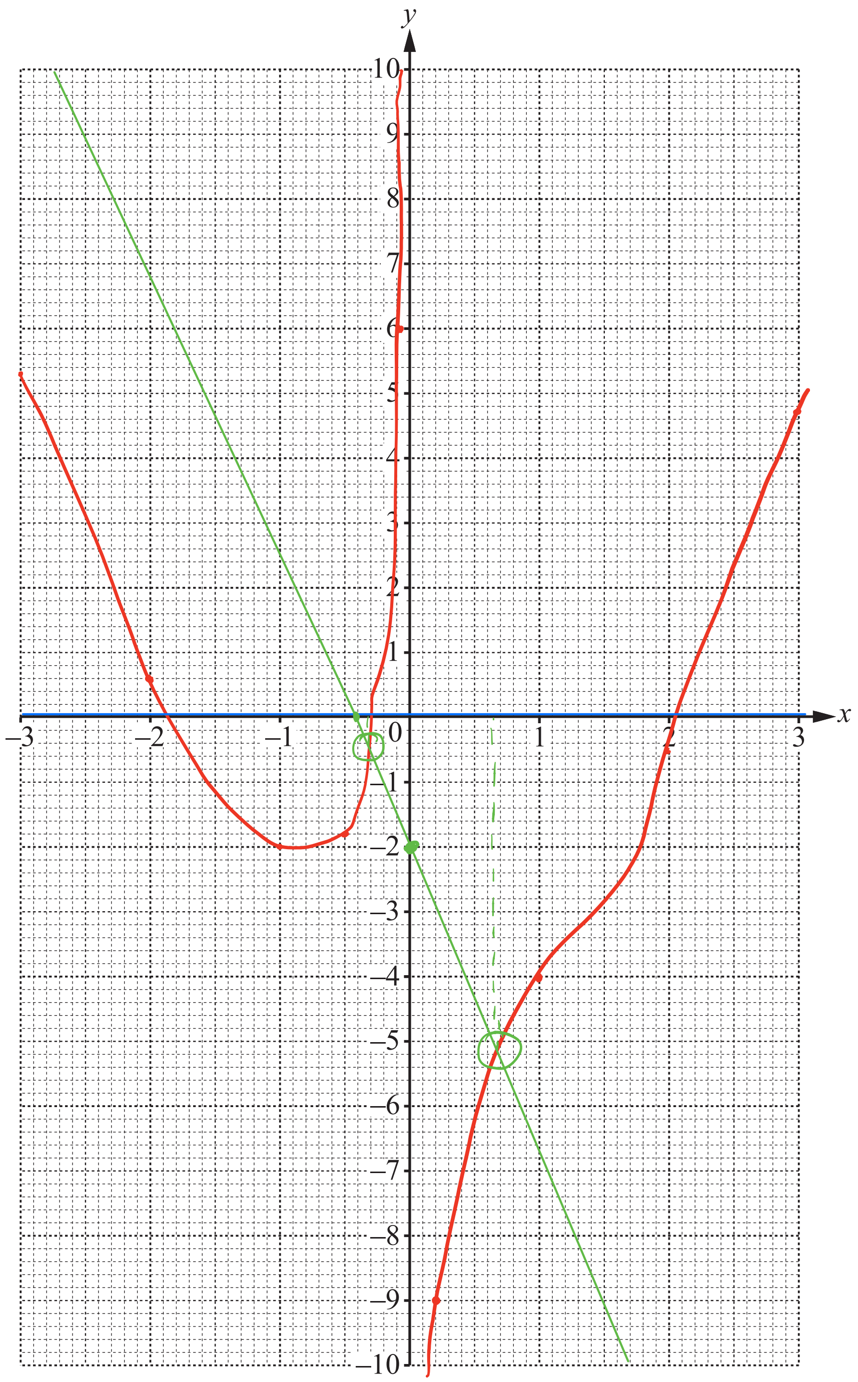
4 $\quad \mathrm{f}(x) = x^2 - \frac{1}{x} - 4\,,\ \ x \neq 0$

(a) **(i)** Complete the table.

x	–3	–2	–1	–0.5	–0.1		0.2	0.5	1	2	3
f(x)	5.3	0.5	-2	–1.8	6.0		–9.0	–5.8	–4	-0.5	4.7

[2]

(ii) On the grid, draw the graph of $y = \mathrm{f}(x)$ for $-3 \leqslant x \leqslant -0.1$ and $0.2 \leqslant x \leqslant 3$.



[5]

(b) Use your graph to solve the equation $f(x) = 0$.

$x =$ ....−1.9.... or $x =$ ....−0.2.... or $x =$ ....2.05.... [3]

(c) Find an integer k, for which $f(x) = k$ has one solution.

any number less than −3

$k =$ ....−4.... [1]

(d) **(i)** By drawing a suitable straight line, solve the equation $f(x) + 2 = -5x$.

$0 = -5x - 2$
$2 = -5x$
$x = \frac{-5}{2} = -0.4$

done in green $f(x) = -5x - 2$

$x = 0 \quad y = 0$
$y = -2 \quad x = -0.4$

$x =$ ....−0.25.... or $x =$ ....0.65.... [4]

(ii) $f(x) + 2 = -5x$ can be written as $x^3 + ax^2 + bx - 1 = 0$.

Find the value of a and the value of b.

$x^2 - \frac{1}{x} - 4 + 2 = -5x$

multiply by x

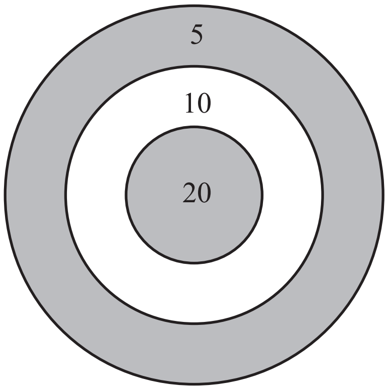
$x^3 - 1 - 4x + 2x = -5x^2$

$x^3 + 5x^2 - 2x - 1 = 0$

$a =$ ....5....

$b =$ ....−2.... [2]

5 Kiah plays a game.
The game involves throwing a coin onto a circular board.
Points are scored for where the coin lands on the board.



If the coin lands on part of a line or misses the board then 0 points are scored.
The table shows the probabilities of Kiah scoring points on the board with one throw.

Points scored	20	10	5	0
Probability	x	0.2	0.3	0.45

(a) Find the value of x.

$x+0.2+0.3+0.45=1$

$x =$ 0.05 [2]

(b) Kiah throws a coin fifty times.

Work out the expected number of times she scores 5 points.

$50 \times 0.3 =$

15 [1]

(c) Kiah throws a coin two times.

Calculate the probability that

(i) she scores either 5 or 0 with her first throw,

$0.45 + 0.3$

0.75 [2]

(ii) she scores 0 with her first throw and 5 with her second throw,

0.45×0.3

0.135 [2]

(iii) she scores a total of 15 points with her two throws.

10 and 5 = 0.2 × 0.3 = 0.06 +

or

5 and 10 = 0.3 × 0.2 = 0.06

= 0.12 .... [3]

(d) Kiah throws a coin three times.

Calculate the probability that she scores a total of 10 points with her three throws.

Outcomes.

0, 0, 10
0, 10, 0
10, 0, 0

3(0.45 × 0.45 × 0.2)

+

Or

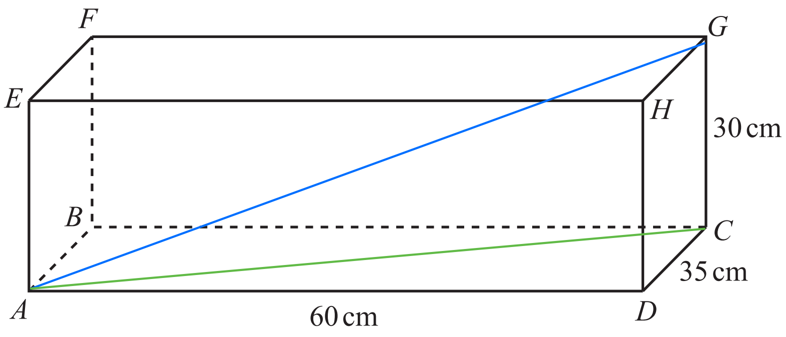
0, 5, 5
5, 0, 5
5, 5, 0

3(0.3 × 0.3 × 0.4)

0.243 .... [5]

3 as there is 3 ways of those outcomes appearing

6 The diagram shows a cuboid.



NOT TO
SCALE

$AD = 60\,\text{cm}$, $CD = 35\,\text{cm}$ and $CG = 30\,\text{cm}$.

(a) Write down the number of planes of symmetry of this cuboid.

3 .... [1]

(b) **(i)** Work out the surface area of the cuboid.

$2(60\times30)+2(60\times35)+2(30\times35) = $ Surface Area.

$= 9900$

9900 .... cm^2 [3]

(ii) Write your answer to **part (b)(i)** in square metres.

cmcm $\xrightarrow{\div100}$ mcm $\xrightarrow{\div100}$ mm

$\dfrac{9900}{(100)(100)} = 0.99$

0.99 .... m^2 [1]

(c) Calculate

(i) the length AG,

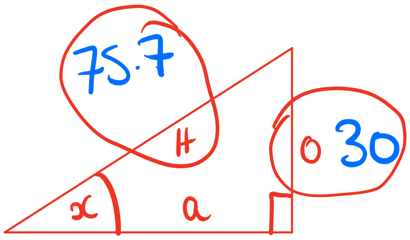
$\sqrt{(60)^2+(35)^2} = 5\sqrt{193}$

$\sqrt{30^2+(5\sqrt{193})^2}$

$AG =$ 75.7 .... cm [4]

(ii) the angle between *AG* and the base *ABCD*.

O
S H C A H T O A



$\sin x = \frac{30}{75.7}$ $\quad x = \sin^{-1}\left(\frac{30}{75.7}\right)$

23.3 .... [3]

(d) **(i)** Show that the volume of the cuboid is 63 000 cm^3.

$(60 \times 30) \times 35 = 63000 \text{ cm}^3.$

[1]

(ii) A cylinder of height 40 cm has the same volume as the cuboid.

Calculate the radius of the cylinder.

Volume of cylinder $= \pi r^2 h$

$h = 40$

$40\pi r^2 = 63000$

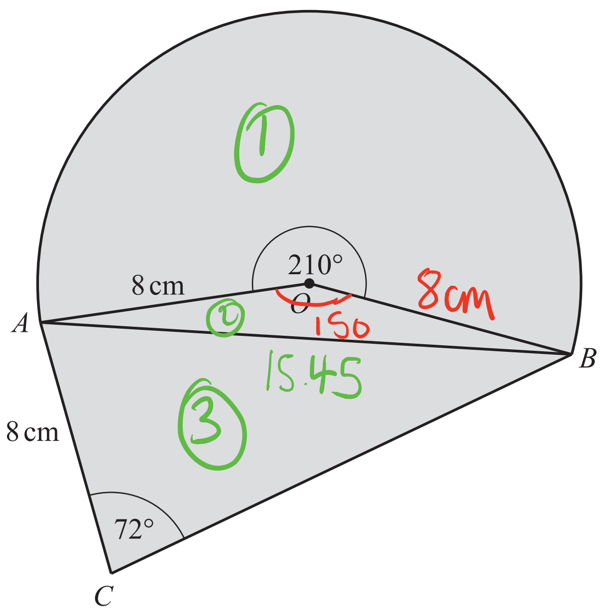
$\pi r^2 = \frac{630\cancel{0}0}{4\cancel{0}}$

$\pi r^2 = 1575$

$r = \sqrt{\frac{1575}{\pi}}$

22.38 .... cm [3]

7



NOT TO SCALE

The diagram shows a design for a logo made from a sector and two triangles.
The sector, centre O, has radius 8 cm and sector angle 210°.
$AC = 8$ cm and angle $ACB = 72°$.

(a) Show that angle $OAB = 15°$.

Isosceles triangle

$150 + 2x = 180$

$x = \frac{30}{2}$ $x = 15$.

[2]

(b) Calculate the length of the straight line AB.

Cos rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 8^2 + 8^2 - 2(8)(8)\cos(150)$

$a = 15.45$

$AB =$ ....15.45.... cm [4]

(c) Calculate angle *ABC*.

Sin rule

$$\frac{\sin(72)}{15.45} = \frac{\sin(x)}{8}$$

$$\frac{8\sin(72)}{15.45} = \sin(x)$$

Angle *ABC* = 29.5 .... [3]

(d) Calculate the total area of the logo design.

① $\frac{210}{360} \times \pi \times 8^2 = 117.2861257$

② $\frac{1}{2} \times 8 \times 8 \times \sin(150) = 16$

③ $\frac{1}{2} \times 8 \times 15.45 \times \sin(78.5) = 60.55934675$

193.8 ....cm² [6]

(e) The logo design is an enlargement with scale factor 4 of the actual logo.

Calculate the area of the actual logo.

Area SF $= 4^2$

$$\frac{193.8}{4^2} = 12.11$$

12.11 ....cm² [2]

8 $\quad f(x) = 5x + 7 \qquad g(x) = \dfrac{4}{x-3}, \; x \neq 3$

(a) Find

(i) fg(1),

$g(1) = \dfrac{4}{1-3} = \dfrac{4}{-2} = -2$

$fg(1) = 5(-2) + 7 = -3$

.... -3 .... [2]

(ii) gf(x),

$\dfrac{4}{(5x+7)-3} = \dfrac{4}{5x+4}$

.... $\dfrac{4}{5x+4}$ .... [2]

(iii) $g^{-1}(x)$,

$y = \dfrac{4}{x-3} = g(x)$

$x = \dfrac{4}{y-3}$

$y - 3 = \dfrac{4}{x}$

$y = \dfrac{4}{x} + 3$

$g^{-1}(x) =$ .... $\dfrac{4}{x} + 3$ .... [3]

(iv) $f^{-1}f(2)$.

$f(2) = 5(2) + 7 = 17$

$f(x) = 5x + 7 = y$

$x = 5y + 7$

$x - 7 = 5y$

$\dfrac{x-7}{5} = y$

$f^{-1}(x) = \dfrac{x-7}{5}$

$f^{-1}(f(2)) = \dfrac{17-7}{5} = 2$

.... 2 .... [1]

(b) f(x) = g(x)

(i) Show that $5x^2 - 8x - 25 = 0$.

$f(x) = g(x) \longrightarrow 5x + 7 = \dfrac{4}{x-3}$

$(5x+7)(x-3) = 4$

$5x^2 - 15x + 7x - 21 = 4$

$5x^2 - 8x - 21 = 4 \longrightarrow 5x^2 - 8x - 25 = 0$

[3]

(ii) Solve $5x^2 - 8x - 25 = 0$.

Show all your working and give your answers correct to 2 decimal places. → Quadratic Formula

$a = 5 \quad b = -8 \quad c = -25$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$

$\dfrac{8 \pm \sqrt{64 - (4 \times 5 \times -25)}}{10} = x$

$\dfrac{8 \pm \sqrt{564}}{10} = x$

x = 3.17 or x = -1.57 [4]

Question 9 is printed on the next page.

$(x_1\ y_1)$ $(x_2\ y_2)$

9 A line joins the points A (–2, –5) and B (4, 13).

(a) Calculate the length AB.

-2 to $4 = 6$

-5 to $13 = 18$

$\sqrt{6^2 + 18^2} = 6\sqrt{10}$

$AB =$ 18.97 [3]

(b) Find the equation of the line through A and B.
Give your answer in the form $y = mx + c$.

$m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{-5-13}{-2-4} = 3 \longrightarrow y = 3x + c$

Point $B(4,13)$ $\quad 13 = 3(4) + c$

$13 - 12 = c = 1$

$y =$ $3x + 1$ [3]

(c) Another line is parallel to AB and passes through the point (0, –5).

Same gradient

where it hits the y-axis.

Write down the equation of this line.

$y = 3x - 5$ [2]

(d) Find the equation of the perpendicular bisector of AB.

gradient $= -\dfrac{1}{m}$

$m = 3$

gradient $= -\dfrac{1}{m} = -\dfrac{1}{3} \longrightarrow y = -\dfrac{1}{3}x + c$

mid-point $= \left(\dfrac{-2+4}{2}, \dfrac{-5+13}{2}\right)$

$= (1, 4)$

(x, y)

$\left.\begin{array}{l} 4 = -\frac{1}{3}(1) + c \\ 4 + \frac{1}{3} = c = \frac{13}{3} \end{array}\right\} y = -\dfrac{1}{3}x + \dfrac{13}{3}$

$y = -\dfrac{1}{3}x + \dfrac{13}{3}$ [5]