Cambridge International Examinations
Cambridge International General Certificate of Secondary Education

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

1049118799

MATHEMATICS **0580/41**

Paper 4 (Extended) **May/June 2016**

2 hours 30 minutes

Candidates answer on the Question Paper.

Additional Materials: Electronic calculator, Geometrical instruments, Tracing paper (optional).

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** questions.
If working is needed for any question it must be shown below that question.
Electronic calculators should be used.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total of the marks for this paper is 130.

The syllabus is approved for use in England, Wales and Northern Ireland as a Cambridge International Level 1/Level 2 Certificate.

This document consists of **19** printed pages and **1** blank page.

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1 **(a)** Kristian and Stephanie share some money in the ratio 3 : 2.
Kristian receives $72.

$24 \times 2 = 2$ parts
$48 =$

(i) Work out how much Stephanie receives.

Kristian = 3parts = 72) ÷3
÷3 (1 part = 24

$ ....48........ [2]

(ii) Kristian spends 45% of his $72 on a computer game.

Calculate the price of the computer game.

$0.45 \times 72 = 32.40$

$ ....32.40........ [1]

(iii) Kristian also buys a meal for $8.40 .

Calculate the fraction of the $72 Kristian has left after buying the computer game and the meal.
Give your answer in its lowest terms.

$32.40 + 8.40 = 40.8$

$\frac{72 - 40.8}{72} = \frac{13}{30}$

........$\frac{13}{30}$........ [2]

(iv) Stephanie buys a book in a sale for $19.20 .
This sale price is after a reduction of 20%.

Calculate the original price of the book.

$80\% = 19.20$

$\frac{19.20}{80} \times 100$

$ ....24........ [3]

(b) Boris invests \$550 at a rate of 2% per year simple interest.

Calculate the amount Boris has after 10 years.

$$SI = \frac{P \times r \times T}{100} = \frac{550 \times 2 \times 10}{100} = 110$$

$$110 + 550 =$$

\$ 660 [3]

(c) Marlene invests \$550 at a rate of 1.9% per year compound interest.

Calculate the amount Marlene has after 10 years.

$$100\% + 1.9\% = 101.9\%$$

$$550 \times \left(\frac{101.9}{100}\right)^{10}$$

\$ 663.90 [2]

(d) Hans invests \$550 at a rate of x% per year compound interest.
At the end of 10 years he has a total amount of \$638.30, correct to the nearest cent.

Find the value of x.

$$550 \times \left(1 + \frac{x}{100}\right)^{10} = 638.30$$

$$\left(1 + \frac{x}{100}\right)^{10} = \frac{638.30}{550}$$

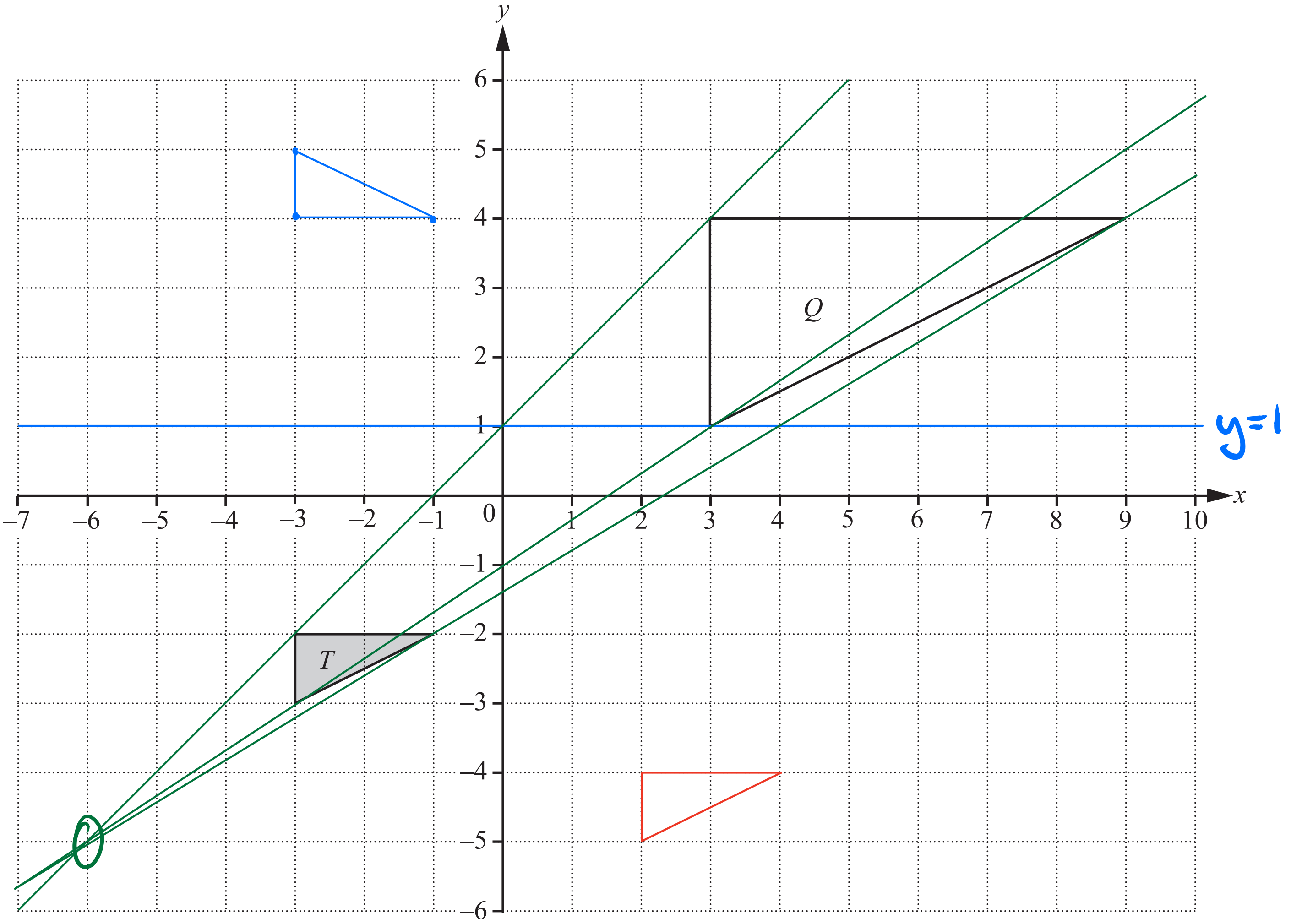
$$1 + \frac{x}{100} = \sqrt[10]{\frac{638.30}{550}}$$

$$\frac{x}{100} = \sqrt[10]{\frac{638.30}{550}} - 1$$

$$x = \left(\sqrt[10]{\frac{638.30}{550}} - 1\right) \times 100$$

$x =$ 1.50 [3]

2 **(a)**



(i) Draw the image of triangle T after a translation by the vector $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$. [2]

(ii) Draw the image of triangle T after a reflection in the line $y = 1$. [2]

(iii) Describe fully the **single** transformation that maps triangle T onto triangle Q.

Enlargement by scale factor 3 and center (-6, -5)

.... [3]

(b) $\mathbf{M} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\mathbf{N} = \begin{pmatrix} 4 & 3 \\ 1 & k \end{pmatrix}$ $\mathbf{P} = \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$

(i) Work out **M** + **P**.

$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}$

$\begin{pmatrix} 2 & 5 \\ 3 & 10 \end{pmatrix}$ [1]

(ii) Work out **PM**.

$\begin{pmatrix} 1 & 3 \\ 0 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 1+9 & 2+12 \\ 18 & 24 \end{pmatrix}$

$\begin{pmatrix} 10 & 14 \\ 18 & 24 \end{pmatrix}$ [2]

(iii) $|\mathbf{M}| = |\mathbf{N}|$

Find the value of k.

$|M| = ad - bc = 4 - 6 = -2$ $K = 1/4$

$|N| = ad - bc = 4K - 3$

so

$4K - 3 = -2$

$4K = 1$

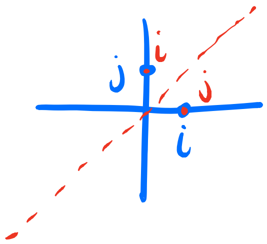
$k =$ .... $1/4$ .... [3]

(c) **(i)** Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Rotation 90° anti-clockwise around the point (0,0).
.... [3]

(ii) Find the matrix which represents a reflection in the line $y = x$.

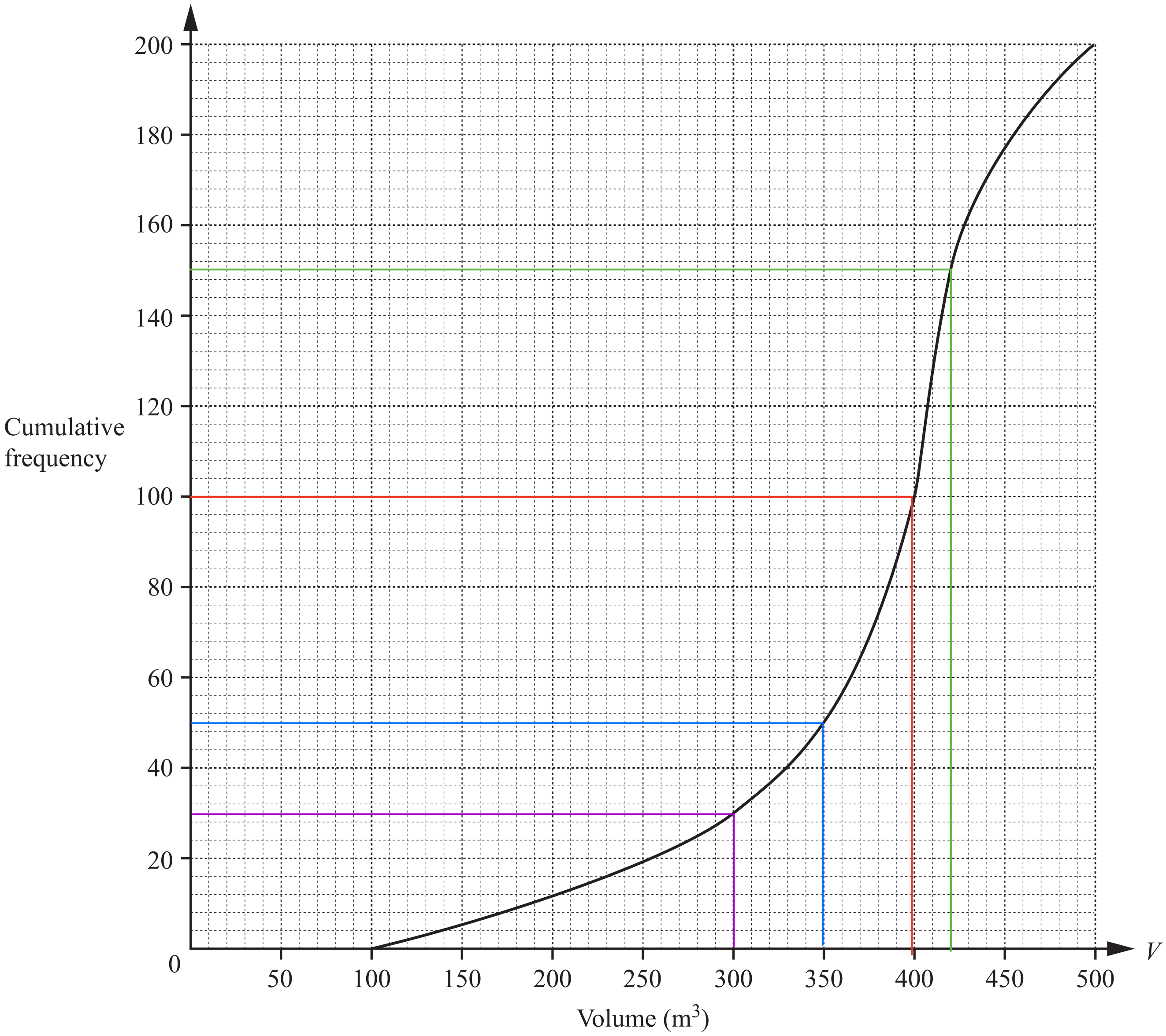
$I = \begin{array}{cc} i & j \\ \end{array}$ $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$



$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ [2]

3 **(a)** 200 students estimate the volume, $V\,\text{m}^3$, of a classroom.
The cumulative frequency diagram shows their results.



Find

(i) the median,

$200/2 = 100$

.......400....... m^3 [1]

(ii) the lower quartile,

$200/4 = 50$

.......350....... m^3 [1]

(iii) the inter-quartile range,

$200 \times \frac{3}{4} = 150 = 420$

$420 - 350$

.......72....... m^3 [1]

(iv) the number of students who estimate that the volume is greater than $300\,\text{m}^3$.

$200 - 30$

.......170....... [2]

(b) The 200 students also estimate the total area, $A\,\text{m}^2$, of the windows in the classroom.
The results are shown in the table.

Area ($A\,\text{m}^2$)	$20 < A \leqslant 60$	$60 < A \leqslant 100$	$100 < A \leqslant 150$	$150 < A \leqslant 250$
F Frequency	32	64	80	24

Total 200

(i) Calculate an estimate of the mean.
Show all your working.

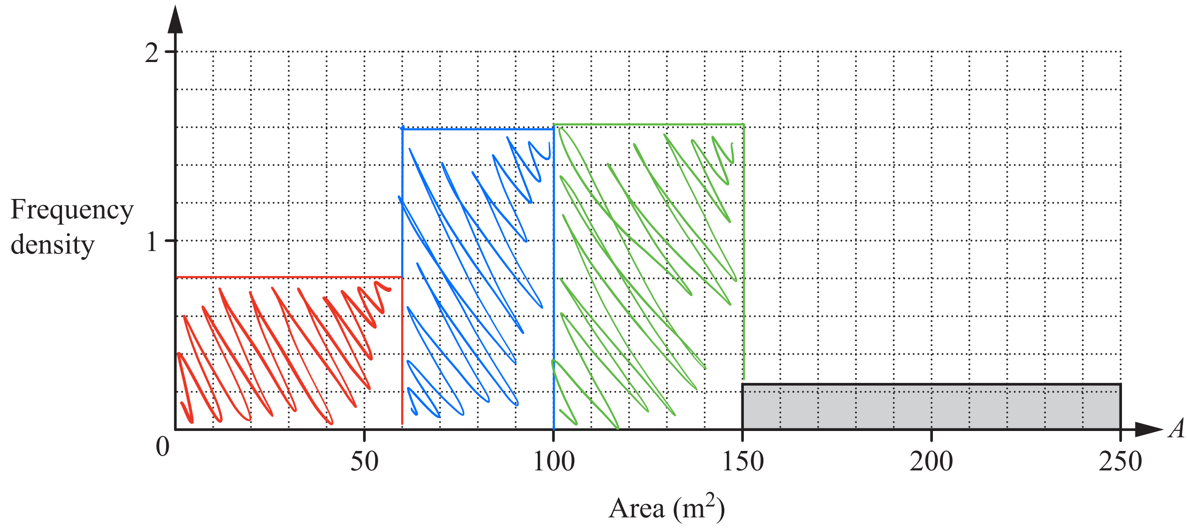
(mid-points) x	40	80	125	200
$F \times x$	1280	5120	10000	4800

Total = 21200

$$\text{Mean} = \frac{\Sigma Fx}{\Sigma F} = \frac{21200}{200}$$

..........106.......... m^2 [4]

(ii) Complete the histogram to show the information in the table.



[4]

(iii) Two of the 200 students are chosen at random.

Find the probability that they both estimate that the area is greater than $100\,\text{m}^2$.

$$\frac{80+24}{200} = \frac{104}{200} = \text{probability}$$

$$\frac{104}{200} \times \frac{103}{199}$$

..........0.27.......... [2]

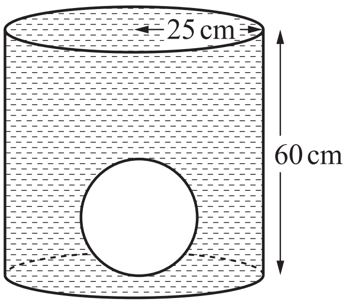
4 **(a)** Calculate the volume of a metal sphere of radius 15 cm and show that it rounds to 14 140 cm^3, correct to 4 significant figures.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

$\frac{4}{3}\pi(15)^3 = 14137.16694 \text{ cm}^3$

$= 14140 \text{ cm}^3$

[2]

(b) **(i)** The sphere is placed inside an empty cylindrical tank of radius 25 cm and height 60 cm.
The tank is filled with water.



NOT TO SCALE

Calculate the volume of water required to fill the tank.

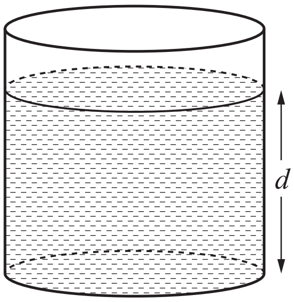
Volume $= \pi r^2 \times h = (25)^2 \times \pi \times 60 = 37500\pi$

$37500\pi - 14140$

$= 103669.7245$

103700 cm^3 [3]

(ii) The sphere is removed from the tank.



NOT TO SCALE

Calculate the depth, d, of water in the tank.

$\pi r^2 d = 103700$

$(25)^2 \times \pi \times d = 103700$

$d = \dfrac{103700}{(25)^2 \times \pi}$

$d = 52.79855713$

$d =$ 52.8 cm [2]

(c) The sphere is melted down and the metal is made into a solid cone of height 54 cm.

(i) Calculate the radius of the cone.
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

$\frac{1}{3}\pi r^2 \times 54 = 14140$

$r^2 = \frac{14140}{54\pi} \times 3$

$r^2 = 250.0500995$

$r = \sqrt{250.0500995}$

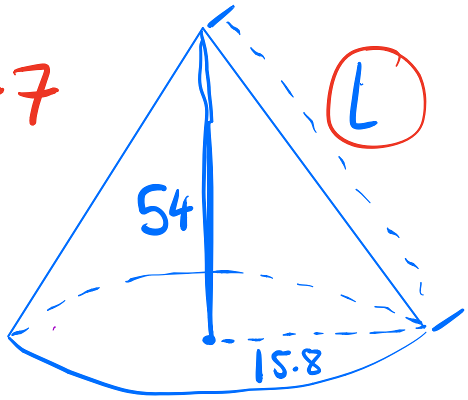
$r = 15.81297251$

$r = 15.8$

15.8 .... cm [3]

(ii) Calculate the **total** surface area of the cone.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi r l$.]

$\sqrt{15.8^2 + 54^2} = 56.26402047$



$\text{Area} = \pi \times 15.8 \times 56.26402047$

$= 2792.786407$

$+ (\pi \times 15.8^2) = 784.26719$

$= 3577.053597$

3577 .... cm^2 [4]

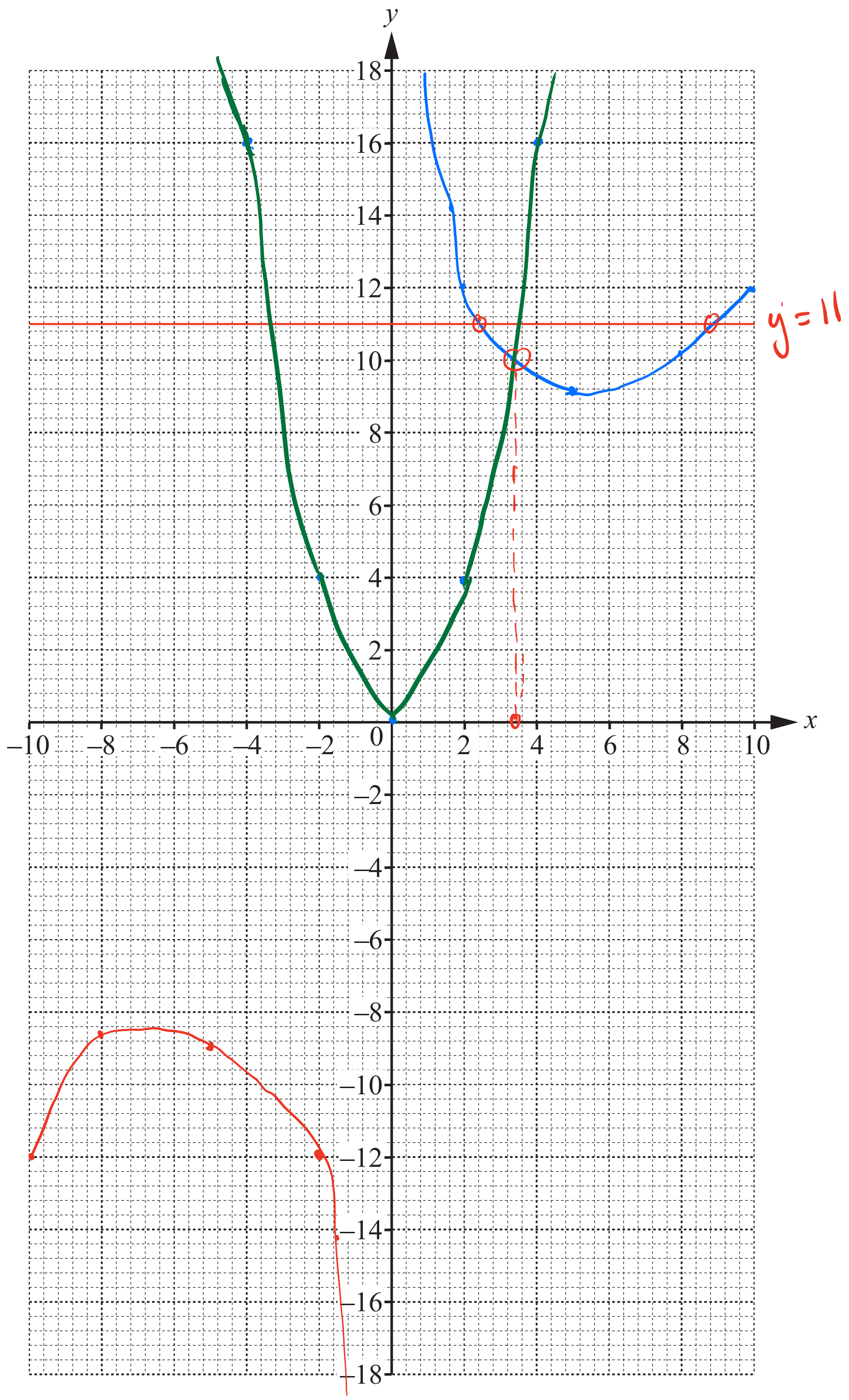
5 $f(x) = \dfrac{20}{x} + x, \quad x \neq 0$

(a) Complete the table.

x	−10	−8	−5	−2	−1.6		1.6	2	5	8	10
f(x)	−12	−10.5	−9	−12	−14.1		14.1	12	9	10.5	12

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-10 \leqslant x \leqslant -1.6$ and $1.6 \leqslant x \leqslant 10$.



[5]

(c) Using your graph, solve the equation $f(x) = 11$.

$x =$ 2.4 or $x =$ 8.8 [2]

(d) k is a prime number and $f(x) = k$ has no solutions.

Find the possible values of k.

2, 3, 5, 7, 9

±2, ±3, ±5, ±7, ±9 [2]

(e) The gradient of the graph of $y = f(x)$ at the point (2, 12) is −4.

Write down the co-ordinates of the other point on the graph of $y = f(x)$ where the gradient is −4.

graph is Symmetrical

(−2 , −12) [1]

(f) **(i)** The equation $f(x) = x^2$ can be written as $x^3 + px^2 + q = 0$.

Show that $p = -1$ and $q = -20$.

$x \times \left(\frac{20}{x} + x = x^2\right) \times x$

$20 + x^2 = x^3$

$x^3 - x^2 - 20 = 0$

$q = -20 \quad p = -1$

[2]

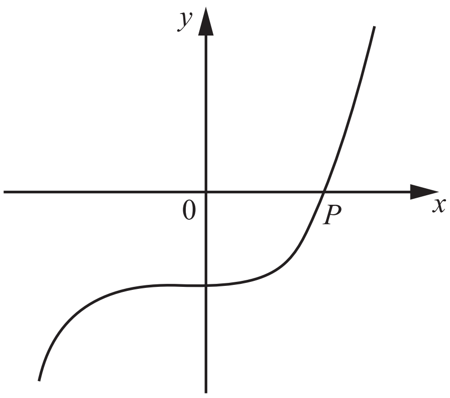
(ii) On the grid opposite, draw the graph of $y = x^2$ for $-4 \leqslant x \leqslant 4$. [2]

(iii) Using your graphs, solve the equation $x^3 - x^2 - 20 = 0$.

Where $x^2 = \frac{20}{x} + x$

$x =$ 3.5 [1]

(iv)



NOT TO SCALE

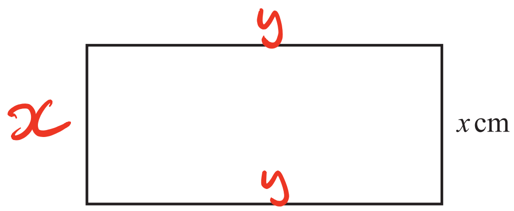
$x^3 - x^2 - 20 = 0$

The diagram shows a **sketch** of the graph of $y = x^3 - x^2 - 20$.
P is the point $(n, 0)$.

Write down the value of n.

$n =$ 3.5 [1]

6 (a)



NOT TO SCALE

The perimeter of the rectangle is 80 cm.
The area of the rectangle is A cm^2.

(i) Show that $x^2 - 40x + A = 0$.

$$\frac{80-2x}{2} = y$$
$$xy = A$$

$$x\left(\frac{80-2x}{2}\right) = A$$
$$x(40-x) = A$$
$$40x - x^2 - A = 0$$
$$x^2 + A - 40x = 0$$

[3]

(ii) When $A = 300$, solve, by factorising, the equation $x^2 - 40x + A = 0$.

$$x^2 - 40x + 300 = 0$$
$$(x-30)(x-10) = 0$$
$$x = 30 \text{ or } x = 10$$

x = ...30.... or x = ...10.... [3]

(iii) When $A = 200$, solve, by using the quadratic formula, the equation $x^2 - 40x + A = 0$.
Show all your working and give your answers correct to 2 decimal places.

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = x$$

$$x^2 - 40x + 200 = 0$$

$$\frac{40 \pm \sqrt{1600 - 800}}{2} = x$$

$$x = \frac{40 \pm \sqrt{800}}{2} = 34.14213562 \text{ or } 5.857864376$$

x = ...34.14.... or x = ...5.86.... [4]

(b) A car completes a 200 km journey with an average speed of x km/h.
The car completes the return journey of 200 km with an average speed of $(x + 10)$ km/h.

(i) Show that the difference between the time taken for each of the two journeys is $\frac{2000}{x(x+10)}$ hours.

$$\frac{200}{x+10} - \frac{200}{x}$$

$$\frac{200x}{x(x+10)} - \frac{200(x+10)}{x(x+10)}$$

$$= \frac{200x - 200x - 2000}{x(x+10)} = \frac{2000}{x(x+10)}$$

[3]

(ii) Find the difference between the time taken for each of the two journeys when $x = 80$.
Give your answer in **minutes** and **seconds**.

$$\frac{2000}{80(90)} = \frac{5}{18} h$$

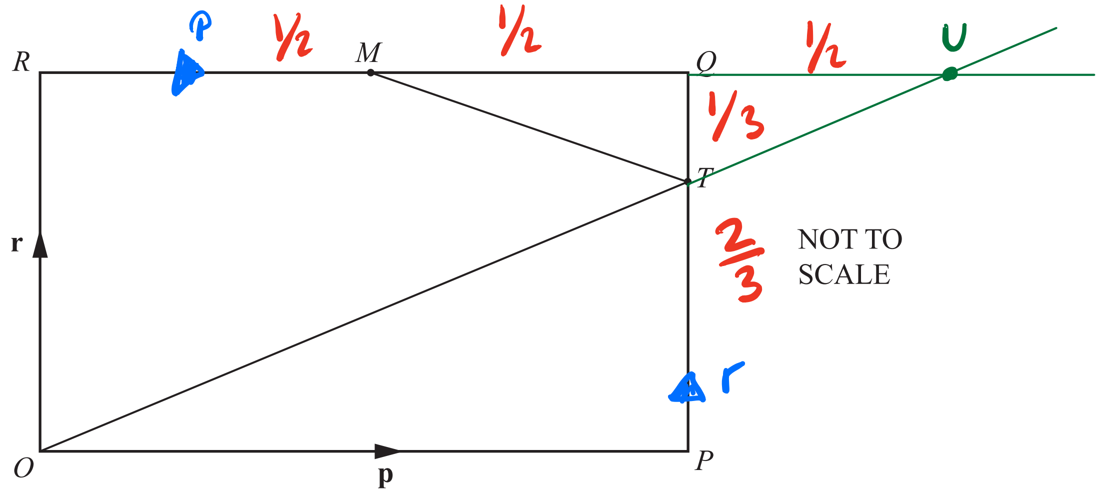
$$\frac{5}{18} \times 60 \times 60 = 1000 \text{ seconds}$$

$$\frac{1000}{60} = 16 \text{ minutes}$$

$$1000 - (16 \times 60) = 40$$

..........16.......... min ....40.......... s [3]

7



OPQR is a rectangle and *O* is the origin.
M is the midpoint of *RQ* and *PT* : *TQ* = 2 : 1.
$\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$.

(a) Find, in terms of **p** and/or **r**, in its simplest form

(i) $\overrightarrow{MQ}$,

$\overrightarrow{MQ}$ = $\frac{1}{2}p$ .... [1]

(ii) $\overrightarrow{MT}$,

$\frac{1}{2}p - \frac{1}{3}r$

$\overrightarrow{MT}$ = $\frac{1}{2}p - \frac{1}{3}r$ .... [1]

(iii) $\overrightarrow{OT}$.

$p + \frac{2}{3}r$

$\overrightarrow{OT}$ = $p + \frac{2}{3}r$ .... [1]

(b) *RQ* and *OT* are extended to meet at *U*.

Find the position vector of *U* in terms of **p** and **r**.
Give your answer in its simplest form.

$\overrightarrow{OU} = \overrightarrow{OR} + \overrightarrow{RU}$

$= r + p + \frac{1}{2}p$

$= \frac{3}{2}p + r$

$\overrightarrow{OU} = \frac{3}{2}p + r$ .... [2]

(c) $\overrightarrow{MT} = \begin{pmatrix} 2k \\ -k \end{pmatrix}$ and $|\overrightarrow{MT}| = \sqrt{180}$.

Find the positive value of k.

$$\sqrt{(2k)^2 + (-k)^2} = \sqrt{180}$$

$$4k^2 + k^2 = 180$$

$$5k^2 = 180$$

$$k^2 = 36$$

$$k = \pm\sqrt{36}$$

k = ...6.. [3]

8 $\qquad f(x) = 2x + 1 \qquad g(x) = x^2 + 4 \qquad h(x) = 2^x$

(a) Solve the equation $\quad f(x) = g(1)$.

$g(1) = 1 + 4 = 5$

$f(x) = 2x + 1$

$2x + 1 = 5$

$2x = 4$

$x =$ ......2...... [2]

(b) Find the value of fh(3).

$h(3) = 2^3 = 8$

$f(8) = 2(8) + 1 = 17$

......17...... [2]

(c) Find $f^{-1}(x)$.

$y = 2x + 1$

$x = 2y + 1$

$\frac{x-1}{2} = y$

hence

$f^{-1}(x) = \frac{x-1}{2}$

$f^{-1}(x) =$ ......$\frac{x-1}{2}$...... [2]

(d) Find gf(x) in its simplest form.

$g(f(x)) = (2x+1)^2 + 4$

$(2x+1)(2x+1) = 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1 + 4$

......$4x^2 + 4x + 5$...... [3]

(e) Solve the equation $h^{-1}(x) = 0.5$.

$h(x) = 2^x$

$2^{1/2} = 1.414213562$

$x =$ ....1.41........ [1]

(f) $\frac{1}{h(x)} = 2^{kx}$

Write down the value of k.

$\frac{1}{2^x} = 2^{kx}$

$k =$ ....-1........ [1]

$1 = 2^x \times 2^{kx}$

$1 = 2^{kx+x}$

$1 = 2^{x(k+1)} \longrightarrow 2^0 = 1$

$k = -1$

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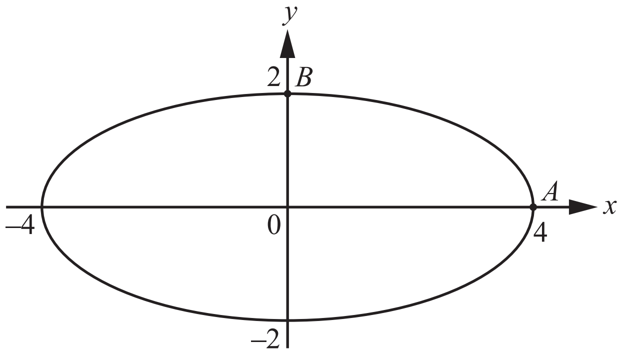


NOT TO SCALE

The diagram shows a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

(a) A is the point (4, 0) and B is the point (0, 2).

(i) Find the equation of the straight line that passes through A and B.
Give your answer in the form $y = mx + c$.

$$\text{gradient} = \frac{0-2}{4-0} = \frac{-2}{4} = \frac{-1}{2}$$

$y = \frac{-1}{2}x + 2$ [3]

(ii) Show that $a^2 = 16$ and $b^2 = 4$.

$A = (4,0)$

$\frac{4^2}{a^2} = 1$

$\frac{1}{a^2} = \frac{1}{4^2}$

$a^2 = 4^2$

$a^2 = 16$

$B = (0,2)$

$\frac{2^2}{b^2} = 1$

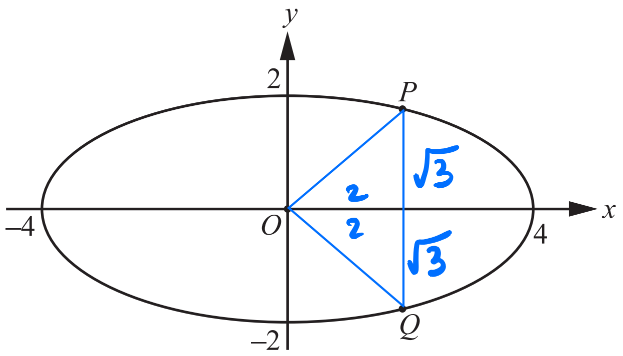
$\frac{1}{b^2} = \frac{1}{2^2}$

$b^2 = 2^2$

$b^2 = 4$

[2]

(b)



NOT TO SCALE

$\frac{4}{16} + \frac{k^2}{4} = 1$

$P\,(2, k)$ and $Q\,(2, -k)$ are points on the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

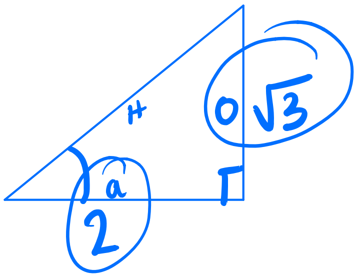
(i) Find the value of k.

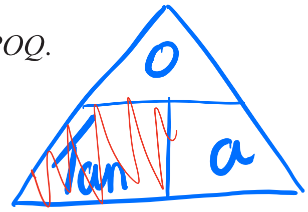
$\frac{4}{16} + \frac{4k^2}{16} = 1$

$4 + 4k^2 = 16$ $\quad 4k^2 = 12$ $\quad k^2 = 3$ $\quad k = \sqrt{3}$

$k =$ .... $\sqrt{3}$ .... [3]

(ii) Calculate angle POQ.





$\tan(x) = \frac{\sqrt{3}}{2}$

$x = 40.89339465$

$x \times 2$

Angle $POQ =$ .... 81.8 .... [3]

(c) The area enclosed by a curve with equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab.

(i) Find the area enclosed by the curve $\frac{x^2}{16} + \frac{y^2}{4} = 1$.

Give your answer as a multiple of π.

$\pi \times a \times b = \pi \times 4 \times 2$

.... 8π .... [1]

(ii) A curve, mathematically similar to the one in the diagrams, intersects the x-axis at $(12, 0)$ and $(-12, 0)$.

Work out the area enclosed by this curve, giving your answer as a multiple of π.

$\frac{12}{4} = 3 =$ Scale Factor

$8\pi \times (3)^2$

.... 72π .... [2]

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