## Functions

## wwu



| 3 | $8 \quad$ (a) <br> (b) | (i) 3 <br> (ii) 4 <br> (iii) $4 x-3$ final answer <br> (iv) $\frac{x+1}{2}$ oe final answer <br> (v) $-\frac{1}{2}$ and $1 \frac{1}{2}$ <br> (i) $y=\frac{16}{x}$ oe <br> (ii) 32 | 1 1 2 2 <br> 4 <br> 2 <br> 1 | M1 for $2(2 x-1)-1$ <br> M1 for $x=2 y-1$ or $\frac{y+1}{2}$ oe or $\frac{f(x)+1}{2}$ oe <br> B1 for $(2 x-1)^{2}$ soi <br> M2 for $2 x-1= \pm 2 \quad$ M1 for $4 x^{2}-2 x-2 x+1$ <br> or M1 for $2 x-1=2$ and M1 for $(2 x+1)(2 x-3)$ <br> or correct substitution <br> in formula <br> soi by $(4 \pm \sqrt{ } 64) / 8$ <br> Condone $y=k / x$ and $k=16$ stated <br> M1 for $y=\frac{k}{x}$ oe |
| :---: | :---: | :---: | :---: | :---: |
| 4 | (b) <br> (c) <br> (d) | (i) 11 <br> (ii) 22 <br> $\frac{x+1}{4}$ oe final answer <br> $16 x^{2}-8 x+7$ final answer <br> 0.5 or $1 / 2$ www | 2 <br> 3 <br> 3 | M1 for $x+1=4 y$ or $\frac{\mathrm{g}(x)+1}{4}$ or $\frac{y+1}{4}$ <br> M1 for $6+(4 x-1)^{2}$ and B1 for $16 x^{2}-4 x-4 x+1$ or better seen <br> M2 for $16 x-4-1=3$ or better or M1 for $4(4 x-1)-1(=3)$ <br> Alt method <br> M2 allow $\mathrm{g}^{-1} \mathrm{~g}^{-1}(3)$ complete method or M1 for $\mathrm{g}(x)=\mathrm{g}^{-1}(3)$ |
| 5 | 7 (a) <br> (b) <br> (c) <br> (d) <br> (e) | $-3,-4.25,-3$ <br> 10 correct points plotted <br> Smooth curve through their 10 points and correct shape <br> Two separate branches <br> (i) 0.7 to 0.85 <br> (ii) Any value of $k$ such that $k \leqslant-3$ and must be consistent with their graph <br> $y=5 x$ drawn <br> -0.6 to $-0.75,0.55$ to 0.65 <br> Tangent drawn at $x=-2$ <br> $y$ change / $x$ change attempt <br> 2.7 to 4.3 | 1, 1, 1 <br> P3ft <br> C1 <br> B1ft <br> 1 1 ft <br> L1 <br> 1, 1 <br> T1 <br> M1 <br> A1 | Allow -4.2 or -4.3 for -4.25 <br> P2ft for 8 or 9 correct <br> P1ft for 6 or 7 correct <br> Correct shape not ruled, (curves could be joined) <br> Indep but needs two 'curves' on either side of $y$ axis <br> - 1 each extra <br> ft consistent with their graph <br> (If curves are joined then $k=-3$ only) <br> Ruled and long enough to meet curves Indep - 1 each extra <br> Must be a reasonable tangent, not chord, no clear daylight <br> Depend on T and uses scales correctly. Mark intention - allow one slight slip e.g. sign error from coords but not scale misread <br> If no working shown and answer is out of range - check their tangent for method <br> Answer in range gets 2 marks after T1 earned |



\begin{tabular}{|c|c|c|c|c|}
\hline 8 \& 4 (a)
(b)

(c)
(d) \& $\left\{\begin{array}{l}4 \\ -5.8 \text { or }-5.75 \text { or }-5.7 \\ -2 \\ 10 \text { correct plots } \mathrm{ft} \\ \\ \text { Correct shape curve through } 10 \text { points } \\ \text { (condone } 2 \text { points slightly missed) } \\ \text { Two separate branches not crossing } y \text {-ax } \\ -2.5 \text { to }-2.3 \\ -0.5 \text { to }-0.4 \\ 2.75 \text { to } 2.9 \\ \text { Correct tangent drawn at } x=-2 \\ -4 \text { to }-2.5\end{array}\right.$ \& 1
1
1
P 3 ft

$\mathrm{C} 1 \mathbf{f t}$
s
B 1

1
1
1

2 \& | ft from their values in (a) generous with $(-0.25,12.1)$ |
| :--- |
| P 2 for 8 or 9 correct plots ft or P1 for 6 or 7 correct plots ft ft their points if shape correct - ignore anything between -0.25 and 0.25 |
| C 1 and B 1 are independent |
| Allow slight daylight |
| Dep on T1 |
| M1 Rise/Tread attempt Dep on T1 or SC1 for answer in range 2.5 to 4 after T1 | <br>

\hline 9 \& | $5 \text { (a) }$ |
| :--- |
| (b) |
| (c) | \& | $9.11,4.25,2, \ldots, 2,4.25,9.11$ |
| :--- |
| 12 points plotted |
| Smooth curve through 12 points Two branches, neither touching $y$-axis |
| (i) $x=0$ |
| (ii) tangent at -1.5 -3 to -1.8 |
| (iii) -1.7 to $-1.55,-0.7$ to -0.55 , 0.55 to $0.7,1.55$ to 1.7 |
| (iv) $y=2 x$ drawn to meet graph twice 1 1.8 to 1.9 | \& | 3 |
| :--- |
| 5 |
| 1 |
| T1 |
| 2 |
| 2 |
| B1 |
| B1 |
| B1 | \& | B2 for 4 or 5 correct and B1 for 2 or 3 correct |
| :--- |
| P3, ft their (a), P2 for 10 or 11 points, P1 for 8 or 9 . |
| C1 correct shape ft their points shape same. Ignore anything between -0.5 and 0.5 . |
| B1 independent |
| Dependent on tangent |
| M1(also dep on T1) for attempt at rise/run or SC1 for 1.8 to 3 |
| B1 for 1 or more correct | <br>

\hline 10 \& $9(\mathrm{a})$
(b)

(c)
(d) \& (i) $\quad 81$
(ii) 8.5
$\frac{x-1}{3} \mathrm{oe}$
$3 x^{2}+12 x+13$ final answer
$(x=) \frac{-3 \pm \sqrt{3^{2}-4(1)(1)}}{2(1)}$

$-2.62,-0.38$ final answer \& | 2 2 |
| :--- |
| 2 |
| 2 |
| 2 |
| 1,1 | \& | B1 for $(f(2)=) 7$ |
| :--- |
| B1 for $(f(0.5)=) 2.5$ |
| M1 for $(x=) \frac{y-1}{3}$ or $(x=) \frac{\mathrm{f}(x)-1}{3}$ or $3 y=x-1$ or $3 \mathrm{f}(x)=x-1$ or -1 then $\div 3$ in flowchart (must be clear) |
| M1 for $3(x+2)^{2}+1$ or better |
| B1 for $\sqrt{3^{2}-4(1)(1)}$ or better Seen anywhere If in form $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}$ oe, B 1 for $p=-3$ and $r=2(1)$ or $\left(x+\frac{3}{2}\right)^{2} \quad$ B1 then $\sqrt{\frac{9}{4}-1}$ B1 If $0, \mathrm{SC} 1$ for -2.6 or -2.62 or $-2.618 \ldots$ and $-0.4(0)$ or -0.38 or -0.382 to -0.381 seen Answers only B1 B1 | <br>

\hline
\end{tabular}

| 11 | 5 | (a) $1,-1,3.5$ <br> (b) 10 correct points plotted <br> Smooth curve through at least 8 points and correct shape <br> (c) $\begin{aligned} & -2.2 \text { to }-2.1 \\ & -0.65 \text { to }-0.45 \\ & 2.5 \text { to } 2.7 \end{aligned}$ <br> (ii) $(k<)-4$ to -3.7 $(k>) 1.7 \text { to } 2$ <br> (d) (i) Ruled line gradient 3 and $y$-intercept -2 over the range -1 to 3.5 <br> (ii) $(a=)-12,(b=) 2$ <br> (iii) 0.1 to 0.2 and 3.3 to 3.4 cao | 1,1,1 <br> P3ft <br> C1ft <br> 1ft <br> 1ft <br> 1ft <br> 1ft <br> 1ft <br> 3 <br> 1,1 <br> 1,1 | P2ft for 8 or 9 correct <br> P1ft for 6 or 7 correct <br> Allow points to be implied from curve <br> Correct cubic shape, not ruled <br> Correct or ft their $x$ values <br> If ft and more than 3 solns then 2 marks maximum <br> Correct or ft their graph for $y$ values at max and min <br> After 0 scored SC1 for both correct but reversed <br> B2 for correct but freehand or short or M1 for a ruled line of gradient 3 or passes through $(0,-2)$ (but not $y=-2$ ) <br> After 0, M1 for $x^{3}-6 x-6 x-2+4(=0)$ or better |
| :---: | :---: | :---: | :---: | :---: |
| 12 | 2 | (a) (i) 39 <br> (ii) $\frac{8}{x}+2$ or $\frac{8+2 x}{x}$ or $\frac{2(4+x)}{x}$ or $8 x^{-1}+2$ final answer <br> (b) -2.5 oe <br> (c) 2.2 oe <br> (d) (i) $4 x-2=\frac{2}{x}+1$ <br> At least 1 intermediate step and $4 x^{2}-3 x-2=0$ <br> (ii) $\frac{-(-3) \pm \sqrt{(-3)^{2}-4(4)(-2)}}{2(4)}$ <br> 1.18 and -0.43 cao | 2 <br> 2 <br> 2 <br> 2 <br> E1 <br> B1 <br> B1 <br> B1B1 | B1 for $(f(2)=) 6$ or $6^{2}$ seen or $(4 x-2)^{2}+3$ seen M1 for $4\left(\frac{2}{x}+1\right)-2$ <br> M1 for $2+x=0.2 x$ oe or $\frac{2}{x}=0.2-1$ or better M1 for $\frac{2}{\frac{5}{3} \text { oe }}+1$ allow 1.66 to 1.67 for $5 / 3$ or $\frac{2}{2 / x+1}+1$ <br> oe with these four terms <br> No errors <br> B1 for $\sqrt{(-3)^{2}-4(4)(-2)}$ or better (41) and in form $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}$ B1 for $-(-3)$ and 2(4) or better SC1 for 1.18 and -0.43 seen or 1.2 and -0.4 or $1.17 \ldots$ and $-0.425 \ldots$ |


| 13 | 7 | (a) $8.7,-3.2,-10$ <br> (b) 6 correct points plotted <br> Smooth curve through 6 points and correct shape <br> (c) Ruled tangent drawn at $x=2$ <br> Rise/run (using correct scales) 3.4 to 4 <br> (d) $k>1.85$ or $k>$ any value greater that 1.85 <br> (e) (i) Correct ruled line for $-3 \leq x \leq 3$ <br> (ii) -1.75 to -1.9 <br> (f) (i) $x^{2}+\frac{1}{x}=x+2$ <br> (ii) $(y=) x+2$ | B3 <br> P2ft <br> C1ft <br> T1 <br> M1 <br> A1 <br> B1 <br> B2 <br> B1 <br> B2 <br> B1ft | 8.66(..) or 8.67, $-3.24,-9.99$ if given to 2 dp <br> B1 for each correct value <br> P1ft for 5 or 4 correct <br> $\mathbf{C 0}$ if curve crosses $y$-axis <br> Not chord, allow slight daylight <br> Dep T1 <br> Accept $\geq$ Ignore $k<$ any value greater than 1.85 <br> SC1 for short ruled line or good freehand complete line or any ruled line grad -1 or ruled with $y$ intercept of $1(\operatorname{not} y=1)$ <br> B1 for $x^{2}-x-2+\frac{1}{x}=0$ oe seen or $1+x^{3}=x^{2}+2 x$ seen or their $a x+b$ numerical $a \neq 0$ and $b \neq 0$ |
| :---: | :---: | :---: | :---: | :---: |
| 14 | 2 | (a) $0.5,4$ <br> (b) 6 points plotted ft Correct shaped curve through 6 poin (exponential) <br> (c) (i) Correct ruled line reaching both points <br> (ii) $6 \div 3$ oe <br> (iii) -0.8 to -0.6 <br> (d) Tangent drawn at (1, 2) <br> Rise/run attempt using correct scales 1.2 to 1.6 cao | $1+1$ <br> P2 <br> C1 <br> L1 <br> 1 <br> 1 <br> T1 <br> M1 <br> A1 | P1 for 5 points <br> Ignore to left of $x=-2$ <br> Allow 'test' with a coordinate on the line (not 0,2) <br> Dep on $\mathbf{L 1}$ <br> Not chord, allow up to 1 mm daylight <br> Dep on $\mathbf{T 1}$ |
| 15 | 8 | (a) 243 <br> (b) $\frac{1-x}{2}$ or $\frac{x-1}{-2}$ final ans <br> (c) $\frac{-1 \pm \sqrt{1^{2}-4(1)(-1)}}{2(1)}$ <br> $-1.62,0.62$ <br> (d) $4 x^{2}-6 x+1$ final ans www3 <br> (e) 9 | 2 <br> 2 <br> B2 <br> B1B <br> 3 <br> 1 | B1 for $(g(-2)=) 5$ seen or $3^{(1-2 x)}$ <br> M1 for $x=1-2 y$ or $x=(1-y) / 2$ <br> B1 for $\sqrt{1^{2}-4(1)(-1)}$ or better $(\sqrt{5})$ seen anywhere <br> If in form $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}$ <br> B1 for $p=-1$ and $r=2(1)$ <br> SC1 for -1.62 and 0.62 seen or -1.6 or -1.618 .. and 0.6 or $0.618 \ldots$ <br> M1 for $(1-2 x)^{2}+(1-2 x)-1$ or better and B1 for $(1-2 x)^{2}=1-2 x-2 x+4 x^{2}$ or better |




| 20 | (b) <br> (c) <br> (d) | (i) $14 \quad-5.5 \quad 20$ <br> (ii) 10 correct points plotted <br> Smooth curve through all 10 points correct shape <br> -4.8 to $-4.6,-0.4$ to $-0.2,3$ to 3.1 www <br> Tangent drawn at $x=-4$ <br> Attempts $y$ step $/ x$ step with correct scales <br> 6 to 11 <br> (i) Ruled line through $(1,15)$ and $(3,-5)$ <br> (ii) 2.5 to 2.7 | $1+1+1$ <br> P3 ft <br> C1 <br> $1+1+1$ <br> T1 <br> M1 <br> A1 <br> 3 <br> 1 | P2 ft for 8 or 9 correct <br> P1 ft for 6 or 7 correct <br> Centre of point must touch line if exact or be in correct square (including boundaries) <br> Within 1 mm radially of potted points. In absence of plot[s], allow curve to imply $\operatorname{plot}[\mathrm{s}]$ <br> No ruled sections <br> After 0 scored, SC1 for $y=2$ soi <br> Penalise first occurrence of co-ord answers in <br> (b) and (d)(ii) <br> Not chord or daylight <br> Dep on $\mathbf{T 1}$ or close attempt at tangent at $x=-4$ <br> Dep on M1 only <br> L2 for short line but correct or freehand full length correct line. <br> L1 for ruled or freehand line through $(0,10)$ (but not $y=10$ ) or for ruled line with gradient -5 <br> isw for extra solns from wrong curve/line |
| :---: | :---: | :---: | :---: | :---: |
| 21 |  | (i) Tangent drawn at $x=2.5$ <br> (ii) 1.55 to 2.2 <br> 1.42 to 1.45 and 2.8 to 2.82 <br> (i) $4.4,2.5,1.5$ <br> (ii) 6 correct points plotted curve through all 6 points and correct shape <br> (iii) 0.75 to 0.9 <br> 1.6 to 1.7 <br> 2.6 to 2.7 | 2dep <br> 1,1 <br> 2 <br> P2ft <br> C1 <br> 1 <br> 1 <br> 1 | reasonable tangent at correct point, no daylight, or chord, crossing $x$-axis between 1.7, 2.0 when extended if necessary <br> Dependent on correct tangent or close attempt at tangent at $x=2.5$ <br> M1dep attempts $y$ step / $x$ step with correct scales <br> B1 for 2 correct values <br> P1ft for 4 or 5 correct plots <br> Smooth curve but last 3 points may be ruled. In absence of plot[s], allow curve to imply plot[s] <br> Solutions may be in any order |



\begin{tabular}{|c|c|c|c|c|}
\hline 23 \& \begin{tabular}{l}
(b) \\
(c) (i) \\
(ii) \\
(iii) \\
(d)
\end{tabular} \& \begin{tabular}{l}
3, \(0.33[3 \ldots], 1\) \\
Correct quadratic curve \\
Correct exponential curve \\
Answer in range \(1.2<x<1.4\) \\
Answer in range \(1.2<x<1.35\) \\
Answer in range \(0.55<x<0.7\) \\
Correct tangent drawn \\
And answer in range \(-2.5<m<-1.5\)
\end{tabular} \& \begin{tabular}{l}
3 \\
3 \\
\\
3 \\
3 \\
\\
\\
\\
\hline 1 \\
1 \\
1 \\
1 \\
\hline
\end{tabular} \& \begin{tabular}{l}
B1 for each correct value \\
B2FT for 7 correct points or \\
B1FT for 5 or 6 correct points \\
B2FT for 7 correct points \\
or \\
B1FT for 5 or 6 correct points \\
Not from a line other than \(y=4\) ( \(\pm 1 \mathrm{~mm}\) ) \\
B1 for correct tangent at \(x=0.5\) \\
B2 for answer in range dep on close attempt at tangent \\
M1 for [-] \(\frac{\text { rise }}{\text { run }}\) used with values soi from tangent, dep on close attempt at tangent or answer in range \(1.5<m<2.5\) \\
or \\
SC1 for close attempt at tangent to exponential curve and answer in the range \(1.6<m<2.2\)
\end{tabular} \\
\hline 24 \& 9 (a) \& \begin{tabular}{l}
\[
\frac{-1 \pm \sqrt{1^{2}-4 \times 1 \times(-3)}}{2}
\] \\
\(-2.30, \quad 1.30\) final answer
\[
4,30,53
\]
\end{tabular} \& 2
2

3 \& | B1 for $1^{2}-4 \times 1 \times(-3)$ or better and if in the form $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}$ then |
| :--- |
| B1 for $p=-1$ and $r=2(1)$ or better |
| B1 B1 |
| SC1 for -2.30 and 1.30 seen or -2.3 or -2.303 to -2.302 and 1.3 or 1.302 to 1.303 |
| or final answer - 1.30 and 2.30 |
| M1 for $(2 x+7)^{2}+(2 x+7)-3$ and |
| B1 for $(2 x+7)^{2}=4 x^{2}+14 x+14 x+$ 49 oe | <br>

\hline
\end{tabular}



\begin{tabular}{|c|c|c|c|c|}
\hline 26 \& \begin{tabular}{l}
5 (a) \\
(b) \\
(c) \\
(d) \\
(e)
\end{tabular} \& \begin{tabular}{l}
\[
-5.04,1.75,0
\] \\
Fully correct curve
\[
\begin{aligned}
\& -1.6 \text { to }-1.5 \\
\& -0.4 \text { to }-0.3 \\
\& 1.8 \text { to } 1.9 \\
\& -2.6 \text { to }-2.5 \text { www } \\
\& -0.4 \text { to }-0.3 \\
\& 1
\end{aligned}
\] \\
3.25 to 4.25 with correct tangent
\end{tabular} \& 3
5

1
1
1
1
1
1

3 \& | B1 for each correct value |
| :--- |
| B3FT for 10 correct plots from their (a) |
| B2FT for 8 or 9 correct plots |
| or B1FT for 6 or 7 correct plots and SC1 for two branches not joined |
| After $\mathbf{0}$ scored, M1 for $y=2 x-2$ drawn |
| B1 for correct tangent |
| B2 for answer in range dep on close attempt at tangent |
| M1dep for $[-] \frac{\text { rise }}{\text { run }}$ used with values soi from tangent, dep on correct or close attempt at tangent | <br>

\hline 27 \&  \& | (i) 1.4 to 1.6 |
| :--- |
| (ii) 1.15 to 1.25 |
| (iii) - 1 |
| (iv) -2.25 to -2.1 |
| -0.9 to -0.75 |
| 2.2 to 2.35 |
| (i) -15 |
| (ii) ${ }_{2}^{1-x}$ or $\frac{1}{2}-\frac{x}{2}$ oe final answer |
| (iii) $-2,2$ |
| (iv) $\quad \frac{1}{8}$ oe nfww | \& | 1 |
| :--- |
| 1 |
| 1 |
| 3 |
| 2 |
| 2 |
| 3 |
| 3 | \& | B2 for 2 correct or $\mathbf{B 1}$ for one correct or B1 for $y=x$ drawn ruled to cut curve 3 times |
| :--- |
| B1 for $[\mathrm{h}(3)=] 8$ seen or M1 for $1-2\left(x^{2}-1\right)$ or better |
| M1 for $2 x=1-y$ or $x=1-2 y$ or better |
| M1 for $x^{2}-1=3$ or better |
| B1 for one answer |
| M2 for $8 x=1$ or $8 x-1=0$ |
| or M1 for $1-2(3 x)[=2 x]$ | <br>

\hline
\end{tabular}



| 29 | 8 | (a) (i) -6 <br> (ii) 2.75 oe <br> (b) $\frac{x-3}{4}$ or $\frac{x}{4}-\frac{3}{4}$ Final answer <br> (c) (i) 5 <br> (ii) $x^{2}+5 x-7=0$ $\frac{-5 \pm \sqrt{5^{2}-4(1)(-7)}}{2(1)} \mathrm{oe}$ <br> 1.14 and -6.14 final answers | 1 <br> 2 <br> 2 <br> 2 <br> B1 <br> B1 <br> B1 <br> B1 <br> B1 | M1 for $[\mathrm{g}(x)=] 0.5$ or $7 / 14$ Or $\left(\frac{7}{x+1}\right)^{2}+5\left(\frac{7}{x+1}\right)$ oe <br> M1 for $y-3=4 x$ or better or $x=4 y+3$ or better or $\frac{y}{4}=\frac{3}{4}+x$ or flowchart with -3 then $\div 4$ <br> M1 for $4 x=23-3$ or $x+\frac{3}{4}=\frac{23}{4}$ or better <br> May be implied by correct values in formula <br> B1 for $\sqrt{5^{2}-4(1)(-7)}$ or better [53] <br> If in form $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}, \mathbf{B} 1$ for -5 and 2(1) or better <br> No recovery of full line unless seen <br> Or SC1 for 1.1 or $1.140 \ldots$ and -6.1 or $-6.140 \ldots$ <br> Or answers -1.14 and 6.14 |
| :---: | :---: | :---: | :---: | :---: |
| 30 | (a) <br> (b) <br> (c) <br> (d) <br> (e) | 2.125 and 2.375 <br> Correct curve <br> Ruled tangent at $x=2$ <br> Gradient from 7.8 to 10.2 <br> 0 and -1.75 to -1.65 and 1.65 to 1.75 $-1.2 \text { to }-0.8<k<2.8 \text { to } 3.2$ | 2 <br> B4 <br> B1 <br> 2 <br> 2 <br> 2 | B1 for one correct value <br> B3FT for 11 correct plots <br> or <br> B2FT for 9 or 10 correct plots <br> or <br> B1FT for 7 or 8 correct plots <br> No daylight at $x=2$. Consider point of contact as midpoint between two vertices of daylight, this must be between $x=1.8$ and 2.2 <br> Dep on B1 awarded <br> Allow integer/integer or a mixed number if within range <br> or <br> M1 dep for (change in $y$ ) $\div($ change in $x$ ) Dependent on any tangent drawn or close attempt at a tangent at any point <br> Must see correct or implied calculation from a drawn tangent <br> B1 for two correct values <br> B1 for each correct or SC1 for reversed answers |



| 32 | (a) <br> (b) <br> (c) <br> (d) <br> (e) <br> (f) | 2 <br> $1-x$ $x^{2}-2 x+2$ <br> $-6$ <br> $\sqrt{(-3)^{2}-4(1)(1)}$ or better <br> $p=-(-3)$ and $r=2 \times 1$ oe <br> $0.38,2.62$ <br> $\mathrm{f}(x)$ and $\mathrm{g}(x)$ | 2 <br> 1 <br> 3 <br> 1 <br> B1 <br> B1 <br> B1B1 | B1 for $g\binom{1}{2}=\frac{1}{2}$ soi or $[\mathrm{fg}=] \begin{gathered}1 \\ 1-x\end{gathered}$ <br> Accept equivalents e.g. $-(x-1)$ <br> M1 for $(1-x)^{2}+1$ <br> B1 for $\left\lfloor(1-x)^{2}=\right\rfloor 1-x-x+x^{2}$ or better <br> or for $\left(x-\frac{3}{2}\right)^{2}$ <br> Must see $\frac{p+\sqrt{q}}{r}$ or $\frac{p-\sqrt{q}}{r}$ or both <br> or for $\frac{3}{2}+o r-\sqrt{\left(\frac{3}{2}\right)^{2}-1}$ <br> SC1 for answers 0.4 and 2.6 or 0.3819 to 0.3820 and $2.618 \ldots$ <br> or 0.38 and 2.62 seen in working or for -0.38 and -2.62 as final ans <br> Accept f and g or $1 / x$ and $1-x$ |
| :---: | :---: | :---: | :---: | :---: |
| 33 | (a) <br> (b) <br> (c) <br> (d) | $0,4.5,3.11[1 \ldots]$ <br> Complete correct curve with minimum below $y=2$ $\begin{aligned} & -0.5 \text { to }-0.6 \\ & 0.6 \text { to } 0.7 \\ & 2.8 \text { to } 2.9 \end{aligned}$ <br> Correct line or no line and -0.7 to - 0.6 nfww | 5 <br> 1 <br> 1 <br> 3 | B1, B1, B1 <br> B3 FT for 9 points correctly plotted <br> B2 FT for 7 or 8 points correctly plotted or B1 FT 5 or 6 points correctly plotted <br> and B1 indep two separate branches not touching or cutting $y$-axis <br> if $0 \mathbf{S C 1}$ for $y=3$ indicated <br> Must check line - not if wrong line B2 for $y=1-x$ ruled correctly <br> or SC1 for ruled line with either gradient -1 or $y$-intercept 1 but not line $\mathrm{y}=1$ or correct freehand line |


|  | (e) (f) | tangent ruled at $x=2$ <br> and <br> 0.62 to 0.8 $\begin{aligned} & \frac{1}{x^{2}}=-x \text { or } 1+x^{3}=0 \\ & 1=-x^{3} \text { or } x^{3}=-1 \\ & x=\sqrt[3]{-1} \end{aligned}$ | 3 <br> M1 <br> M1 <br> A1 | Accept integer/integer provided in range B1 for correct tangent drawn <br> and M1 for change in $y /$ change in $x$ dep on any tangent or close attempt at tangent at any point Must see correct or implied calculation from a drawn tangent dep M1 <br> dep M2 |
| :---: | :---: | :---: | :---: | :---: |
| 34 | (a) <br> (b) <br> (c) <br> (d) <br> (e) | $4-6 x$ final answer <br> $9 x-8$ final answer <br> $\frac{1}{27}$ final answer <br> $\frac{4-x}{3}$ oe final answer <br> $\frac{4}{3}$ or $1 \frac{1}{3}$ or 1.33 or better | 1 <br> 2 <br> 3 <br> 2 <br> 3 | M1 for $4-3(4-3 x)$ seen <br> M2 for $3^{-3}$ soi by final answer $0.037037 \ldots$ to 3 sf or better or M1 for $[g(-1)=] 3$ soi <br> M1 for a correct first step $3 x=4-y$ oe or $x=4-3 y$ or $\frac{y}{3}=\frac{4}{3}-x$ <br> M2 for $3 x-4=0$ or better <br> or M1 for $3^{-(4-3 x)}$ |
| 35 | 5 (a) <br> (b) <br> (c) | $-2,5.5$ <br> Correct curve $\begin{aligned} & -2.6 \leqslant x \leqslant-2.4 \\ & 0.6 \leqslant x \leqslant 0.7 \\ & 1.8 \leqslant x \leqslant 1.9 \end{aligned}$ | 2 <br> 5 <br> 3 | B1 for each value <br> B5 for correct curve over full domain or <br> B3FT for 9 or 10 points or B2FT for 7 or 8 points or B1FT for 5 or 6 points Point must touch line if exact or be in correct squar not exact (including boundaries) <br> and <br> B1 independent for one branch on each side of the $y$-axis and not touching or crossing the $y$-axis <br> SC4 for correct curve with branches joined <br> B1 for each value <br> If $\mathbf{B 0}$ then $\mathbf{S C} 1$ for $y=5$ used |



| 37 | $6 \quad$ (a) <br> (b) <br> (c) <br> (d) <br> (e) | $-3,7.375,8.875$ <br> Correct curve <br> (i) Any integer less than 7 or greater than 10 <br> (ii) 7,8 or 9 <br> $y=15 x+2$ ruled and fit for purpose <br> -1.45 to -1.35 and 0.4 to 0.5 <br> Tangent ruled at $x=1.5$ <br> 7 to 12 | $1,1,1$ <br> 4 <br> 1 <br> 1 <br> B2 <br> B2 <br> B1 | Accept 7.4 or 7.37 or 7.38 for 7.375 and 8.9 or 8.87 or 8.88 for 8.875 <br> B3FT for 8 or 9 correct plots <br> B2FT for 6 or 7 correct plots <br> B1FT for 4 or 5 correct plots <br> Point must touch line if exact or be in correct square if not exact (including boundaries) <br> B1 for short line but correct or freehand full length correct line or for ruled line through $(0,2)$ (but not $y=2$ ) or for ruled line with gradient 15 (acc $\pm 1 \mathrm{~mm}$ vertically for 1 horizontal unit) <br> B1 for each <br> No daylight at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=1.4$ and 1.6 <br> Dep on B1 or close attempt at tangent at $x=1.5$ M1 for $y$-step $/ x$ - step for their tangent |
| :---: | :---: | :---: | :---: | :---: |
| 38 | (b) <br> (c) <br> (d) | (i) $(1,2)$ <br> (ii) $y=3 x-1$ cao final answer <br> (i) $(x+5)(x-2)$ isw solutions <br> (ii) $\begin{array}{ll} {[a=]} & -5 \\ {[b=]} & 2 \\ {[c=]} & -10 \end{array}$ <br> (iii) $x=-1.5$ <br> Inverted parabola <br> $x$-axis intercepts at -2 and 9 <br> $y$-axis intercept at 18 <br> (i) $\begin{aligned} & p=6 \\ & q=43 \end{aligned}$ <br> (ii) - 43 | 1+1 <br> 3 <br> 2 <br> 3FT <br> 1FT <br> B1 <br> B2 <br> B1 <br> 3 <br> 1FT | M1 for gradient $=\frac{8--4}{3-1}$ oe and M1 for substituting $(3,8)$ or $(-1,-4)$ into their $y=3 x+\mathrm{c}$ or for finding $y$-intercept is -1 <br> SC1 for $(x+a)(x+b)$ where $a b=-10$ or $a+b=3$ <br> B1FT for each of their 5 and their -2 from (b)(i) and $\mathbf{B 1}$ for $\mathrm{c}=-10$ <br> FT $x=($ their $(a+b)) / 2$ <br> B1 for each <br> After B0 allow SC1 for $(9-x)(2+x)$ oe <br> B2 for $(x+6)^{2}-43$ or $p=6$ or $q=43$ <br> or M1 for $(x+6)^{2}$ or $x^{2}+p x+p x+p^{2}$ <br> and <br> M1 for $-7-(\text { their } 6)^{2}$ or $p^{2}-q=-7$ or $2 p=12$ <br> FT - their $q$ |




| 41 | (b) <br> (c) <br> (d) <br> (e) | 1 3 <br> 2.5 <br> Fully correct graph $-2.6 \text { to }-2.4$ <br> Correct ruled line fit for purpose -1.6 to -1.5 <br> Correct tangent and $0.9 \leqslant \operatorname{grad} \leqslant 1.5$ | 1 1 1 <br> 5 <br> 1 <br> 2 <br> 1 <br> 3 | B3FT for 11, 12 points correct or B2FT for 9,10 correct points or B1FT for 7,8 correct points <br> B1 for branch each side of $y$-axis and not touching $y$-axis <br> SC4 for correct graph but branches joined <br> $\mathbf{S C} 1$ for ruled line through $(0,1)$ but not $y=1$ or ruled line with gradient -1 or for correct line but freehand <br> Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=-3.4$ and -2.6 <br> B2 if close attempt at correct tangent and answer in range (may be small amount of daylight) <br> or B1 for ruled tangent at $x=-3$ within tolerance, no daylight at the point of contact <br> and M1 (dep on B1 or close attempt at tangent) for a tangent at any point and $\frac{\text { rise }}{\text { run }}$ used |
| :---: | :---: | :---: | :---: | :---: |
| 42 | $10 \text { (a) }$ <br> (b) <br> (c) <br> (d) | 9 <br> $4 x^{2}-2 x$ or $2 x(2 x-1)$ final answer <br> $\frac{x+1}{2}$ oe final answer $\frac{4 x+4}{x(x+2)} \text { or } \frac{4 x+4}{x^{2}+2 x} \text { or } \frac{4(x+1)}{x(x+2)}$ <br> or $\frac{4(x+1)}{x^{2}+2 x}$ final answer | 2 <br> 3 <br> 2 <br> 4 | B1 for $[f(3)=] 5$ or $2(2 x-1)-1$ <br> M1 for $(2 x-1)^{2}+(2 x-1)$ <br> B1 for $\left[(2 x-1)^{2}=\right] 4 x^{2}-2 x-2 x+1$ <br> or $(2 x-1)(2 x-1+1)$ <br> M1 for $x=2 y-1$ or $y+1=2 x$ <br> or $\frac{y}{2}=x-\frac{1}{2}$ <br> B1 for $x(x+2)$ oe isw as common denominator <br> B2 for $4 x+4$ as numerator or $\mathbf{B 1}$ for $2(x+2)+2 x$ or better as numerator |


| 43 | 2 (a) <br> (b) <br> (c) | (i) 1.62 or $1.62 \ldots$ <br> (ii) 4 <br> (iii) 7 <br> (iv) $\frac{1}{3}$ oe <br> (i) 0.25 oe and 1 <br> (ii) Correct curve <br> (iii) 2.3 <br> (iv) $y=3 x-1$ oe 3 term equation <br> (v) -1.7 to - 1.5 and 2 | 1 <br> 1 <br> 1 <br> 1 <br> 1 <br> 2 <br> 4 <br> 1FT <br> 3 <br> 2 | B1 for each <br> B3FT for 6 or 7 correct plots or B2FT for 4 or 5 correct plots or B1FT for 2 or 3 correct plots <br> Correct or FT where $y=5$ on their graph <br> B2 for $3 x-1$ or $y=3 x[+c]$ oe <br> or for $m=3$ and $c=-1$ <br> or M1 for [gradient $=$ ] $\frac{8-2}{3-1}$ oe soi by $3 x$ <br> and M1 for substitution of $(1,2)$ or $(3,8)$ into their $y=m x+c$ <br> B1 for either or M1 for $y=x+2$ seen or drawn |
| :---: | :---: | :---: | :---: | :---: |
| 44 | (a) <br> (b) <br> (c) <br> (d) | $\begin{array}{llll}0 & 4 & 0.625 & 0.875\end{array}$ <br> Fully correct smooth curve <br> line $y=x+1$ ruled <br> and <br> 0.2 to 0.3 <br> and <br> 1.8 to 1.95 <br> Tangent ruled at $x=-1.5$ <br> 2.2 to 5 | $1,1,1,1$ <br> 4 <br> 3 <br> B1 <br> 2 | B3 FT for 8 or 9 points <br> or B2 FT for 6 or 7 points or B1 FT for 4 or 5 points <br> Line must be fit for purpose ie at least from $x=0$ to $x=2$ <br> B2 for correct line and 1 correct value or B1 for correct line or SC1 for no/wrong line and 2 correct values <br> No daylight between tangent and curve at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=-1.6$ and $x=-1.4$ <br> dep on B1 <br> M1 for $\frac{\text { rise }}{\text { run }}$ also dep on any tangent drawn or close attempt at tangent at any point Must see correct or implied calculation from a drawn tangent |



| 47 | (a) <br> (b) <br> (c) <br> (d) | $20-22$ <br> smooth correct curve through correct points <br> line $y=\frac{1}{2}(x+1)$ ruled and $\begin{aligned} & -2.85 \text { to }-2.95 \\ & -1 \\ & 0.85 \text { to } 0.95 \end{aligned}$ <br> tangent ruled <br> -1.1 to -1.5 | 3 <br> 4 | B2 for 3 correct <br> B1 for 2 correct <br> B3FT for 8 or 9 correct plots <br> B2FT for 6 or 7 correct plots <br> B1FT for 4 or 5 correct plots <br> FT their table <br> Line must be fit for purpose <br> B3 for correct line and 2 correct values or $\mathbf{B} 2$ for correct line and 1 correct value or B1 for correct line or SC2 for no/wrong line and 3 correct values or SC1 for no/wrong line and 2 correct values <br> No daylight between tangent and curve at point of contact. Consider point of contact as midpoint between two vertices of daylight, the midpoint must be between $x=-1.85$ and $x=-1.65$ <br> dep on B1 <br> M1 for rise/run also dep on any tangent drawn or close attempt at tangent at any point Must see correct or implied calculation from a drawn tangent <br> Accept M1 for answer in range 1.1 to 1.5 after B1 |
| :---: | :---: | :---: | :---: | :---: |
| 48 | $7 \quad$ (a) <br> (b) <br> (c) | 3.5[0] 1.943 .11 <br> Fully correct curve <br> -0.7 to -0.6 | 3 | B1 for each <br> B3 FT for 10 or 11 points or B2 FT for 8 or 9 points or B1 FT for 6 or 7 points <br> B1 indep two separate branches not touching or cutting $y$-axis <br> SC4 for correct curve, but branches joined |







| 56 | 4(a) <br> 4(b) <br> 4(c) <br> 4(d) <br> 4(e) | 10, 7 <br> Correct curve <br> -1.7 to -1.55 <br> Tangent ruled at $x=3.5$ <br> 6.5 to 11 $\begin{aligned} & \text { line } y=2 x+10 \text { ruled } \\ & \frac{\text { AND }}{-1.3} \text { to }-1.1 \\ & 1 \\ & 4.1 \text { to } 4.25 \end{aligned}$ | $\begin{array}{r}2 \\ 4 \\ 4 \\ 1 \\ \text { B1 } \\ \hline\end{array}$ | B1 for each value <br> B3 FT for 10 or 11 correct points <br> B2 FT for 8 or 9 correct points <br> B1 FT for 6 or 7 correct points <br> FT their table <br> FT their graph if one answer <br> No daylight between tangent and curve at point of contact <br> dep on tangent drawn or close attempt at tangent at $x=3.5$ <br> M1 for rise/run also dep on tangent or close attempt at $x=3.5$ <br> B3 for correct line (could be short) and 1 correct value <br> or $\mathbf{B 2}$ for correct line (could be short) <br> or B1 for $[y=] 2 x+10$ seen <br> If zero scored, $\mathbf{S C 1}$ for no/wrong line and 3 correct values |
| :---: | :---: | :---: | :---: | :---: |
| 57 | 7(a) <br> 7(b) <br> 7(c)(i) <br> 7(c)(ii) <br> 7(d) <br> 7(e) | $x=0$ <br> Tangent ruled at $x=0.5$ $-9 \text { to }-6.5$ <br> $0 \quad 2.4$ or better 4 <br> Correct smooth curve <br> $x^{3}+3 x+4=10-8 x^{2}$ and correctly completed <br> line $y=-2 x+2$ drawn and -0.45 to -0.35 nfww | 1 <br> B1 <br> 2 <br> 3 <br> 4 <br> 1 <br> 3 | No daylight between tangent and curve at point of contact <br> dep on ruled tangent or close attempt at tangent at $x=0.5$ <br> M1 for rise/run also dep on tangent or close attempt at tangent at $x=0.5$ <br> B1 for each <br> B3FT for 6 or 7 correct plots or B2 FT for 4 or 5 correct plots or B1 FT for 2 or 3 correct plots <br> FT their table <br> B2 for ruled $y=-2 x+2$ or $\mathbf{B 1}$ for $-2 x+2$ seen or for line $y=-2 x+c$ drawn or for $y=c x+2$ $(c \neq 0)$ drawn and B1 for -0.45 to -0.35 nfww |




| 60 | 6(a) <br> 6(b) <br> 6(c) <br> 6(d) <br> 6(e) <br> 6(f) <br> 6(g) | 256 <br> 8 $9 x^{2}+12 x+5$ <br> 16 <br> $\frac{x-2}{3}$ oe final answer <br> $\frac{4 x^{2}+2 x+1}{3 x+2}$ final answer <br> 16 |  | M1 for $3\left(x^{2}+1\right)+2$ or for $3(2)+2$ <br> M1 for $(3 x+2)^{2}+1$ <br> B1 for $\left[(3 x+2)^{2}=\right] 9 x^{2}+6 x+6 x+4$ oe <br> M1 for $3 x+2=7^{2}+1$ or better <br> M1 for $x=3 y+2$ or for $y-2=3 x$ or for $\frac{y}{3}=x+\frac{2}{3}$ <br> B1 for $x^{2}+1+x(3 x+2)$ or better seen M1 for common denominator $3 x+2$ |
| :---: | :---: | :---: | :---: | :---: |
| 61 | $\begin{gathered} 10(\mathrm{a})(\mathrm{i}) \\ 10(\mathrm{a})(\mathrm{ii} \\ 10(\mathrm{~b}) \end{gathered}$ | $x+5$ <br> $2 \sin x$ oe <br> tangent ruled at $P$ <br> 1.3 to 1.4 | $\begin{array}{r} 2 \\ 2 \\ \text { B1 } \\ \text { B2 } \end{array}$ | B1 for linear equation with positive gradient or intercept 5 <br> B1 for recognition of $\sin$ or $\cos (x-90)$ <br> dep on tangent drawn <br> M1 for rise/run |
| 62 | 5(a)(i) <br> 5(a)(ii) <br> 5(a)(iii) | 2.7 to 2.8 <br> tangent ruled at $x=-2$ $6 \text { to } 10$ $y=2 x-2 \text { ruled }$ <br> and $x=-2.9$ to -2.8 cao | 1 <br> B1 <br> 2 <br> 3 | dep on B1 or a close attempt at tangent at $x=-2$ <br> or M1 for rise/run for their tangent, or close attempt, at any point <br> Must see correct or implied calculation from a drawn tangent <br> After M0, SC1 for gradient of tangent (or close attempt) in range embedded in $y=m x+c$ <br> B2 for correct ruled line <br> or B1 for short line or for freehand line or broken line or ruled line with gradient 2 or with $y$-intercept at $-2($ but not $y=-2)$ |



