



# Linear Programming

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1	10	(a) $20x + 100y \leq 1200$ (b)(i) $x + y \geq 40$ (ii) $y \geq 2$ (c) $x + y = 40$ cao $y = 2$ cao  Required region only region left not shaded or otherwise clearly indicated cao (d) 5 cao (e) 50 cao, 2 cao 270 ft	1 1 1 L1 L1 R2  1 2 1ft	Each line ruled and long enough to enclose required region. If <b>L0, SC1</b> if freehand but otherwise accurate and enclose region <b>SC1</b> if one boundary error – see diagrams  <b>B1 B1</b> ft $5 \times$ their $x + 10 \times$ their $y$
2	9	(a) $x \geq 3$ $y \geq 2$ (b) $x + y \leq 9$ (c) $6x + 14y \leq 84$ (d) $x = 3$ $y = 2$ $x + y = 9$  Line from (0, 6) to (14, 0) Correct quadrilateral unshaded or clearly indicated (e) \$ 70	1, 1 1 1 1, 1 2 2 1 2	Accept clear and freehand lines long enough to define the correct quadrilateral SC1 for line through (0, 9) or (9, 0)  B1 for through (0, 6) or (14, 0)  B1 for considering (7, 2)
3	8	(a) (i) There are up to 5 large coaches oe (ii) $50x + 30y \geq 300$ oe  (b)  $x = 5$ ruled $x + y = 10$ ruled $5x + 3y = 30$ ruled  Correct region indicated cao (c) (i) 5 2 (ii) 2950	1  <b>E2</b>   L1 L1 L2  R1  1 1 1ft	E.g. can't hire more than 5 large coaches The maximum is 5 large coaches The large coaches are less than or equal to 5  No errors Allow in words provided clear e.g. 50 in large coaches and 30 in small coaches must equal 300 seats or more <b>M1</b> for associating 50 with $x$ or large coaches and 30 with $y$ or small coaches  Freehand lines – 1 pen once. All lines must be long enough to make full boundary of their region accept dashed or solid lines  <b>L1</b> for ruled line with intercepts at (0, 10) or (6, 0) within 2mm by eye at intercepts (extend if line is short)  <b>R1</b> Allow if slight inaccuracy(s) in diagonal lines Allow any clear indication of region  After 5 and 2 in working ignore attempts to calculate costs  ft their $5 \times 450 +$ their $2 \times 350$ provided positive integers

4	10	<p>(a) <math>20x + 10y \geq 200</math></p> <p>(b) <math>x + y \leq 15, y \geq 3, y \leq x</math></p> <p>(c)</p> <p><math>2x + y = 20</math> ruled</p> <p><math>x + y = 15</math> ruled</p> <p><math>y = x</math> ruled</p> <p><math>y = 3</math> ruled</p> <p>Quadrilateral identified</p> <p>(d) (i) 47 cao</p> <p>(ii) 7, 6 cao</p>	<p>1 In (a), (b) –1 once for wrong symbol</p> <p>3 B1 for each</p> <p>All lines long enough to make full boundary of region, accept dashed or solid lines, 2 mm acc at intercepts</p> <p>B2 B1 for ruled line through (10, 0) or (0, 20)</p> <p>B1</p> <p>B1</p> <p>B1 –1 once, freehand</p> <p>R1 Allow if slight inaccuracy(s) in diagonal lines</p> <p>Allow any clear indication of region</p> <p>1</p> <p>2 M1 for any <math>5x + 2y</math> in their region evaluated to equal their 47</p>	
5	3	<p>(a) (i) <math>x &gt; 4</math></p> <p>(ii) <math>y &gt; 9</math></p> <p>(iii) <math>x + y &lt; 20</math></p> <p>(b) <math>5x + 10y &lt; 170</math> seen</p> <p>(c) (i) <math>x = 4</math> ruled</p> <p><math>y = 9</math> ruled</p> <p><math>x + y = 20</math> ruled</p> <p><math>x + 2y = 34</math> ruled</p> <p>Correct region indicated cao</p> <p>(ii) 145 cao (from 11, 9) www 2</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p> <p>2</p> <p>1</p> <p>2</p>	<p>Each line long enough to enclose their region</p> <p>Condone good freehand or dotted</p> <p><math>y = 9</math> must be <b>between</b> 8.8 and 9.2</p> <p>B1 for gradient = –1 or <math>y</math> intercept = 20 or <math>x</math> intercept = 20. Exclude lines parallel to either axis.</p> <p>B1 for <math>y</math> intercept = 17 or <math>x</math> intercept = 34. Exclude lines parallel to either axis.</p> <p><b>Dependent</b> on all 6 marks for the 4 lines.</p> <p>M1 for using <math>5x + 10y</math> when <math>x + y = 20</math> and integers <math>(x, y)</math> is in their region</p>

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6	7	<p>(a) (i) <math>x \geq 5</math></p> <p><math>y \leq 8</math></p> <p><math>x + y \leq 14</math></p> <p><math>y \geq \frac{1}{2}x</math> oe</p> <p>(ii) <math>x = 5</math> ruled  <math>y = 8</math> ruled  <math>x + y = 14</math> ruled  <math>y = \frac{1}{2}x</math> ruled  region indicated</p> <p>(b) (i) 480</p> <p>(ii) 6, 8</p>	<p>4</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1dep</p> <p>2</p> <p>1</p>	<p><b>B1</b> for each correct inequality</p> <p>Penalise the first occurrence only when strict inequalities used</p> <p>Each line long enough to be boundary of region</p> <p>Check at intercepts</p> <p>Check at (10, 5)</p> <p><b>Dependent</b> on 4 lines correct</p> <p><b>M1</b> for <math>20 \times x + 45 \times y</math> where <math>x</math> and <math>y</math> are integers and <math>(x, y)</math> is in their quadrilateral</p> <p>In correct order</p>
7	3	<p>(a) (i) <math>x \geq 5</math></p> <p>(ii) <math>y \geq 11</math></p> <p>(iii) <math>x + y \geq 20</math></p> <p>(b) <math>4x + 8y \leq 160</math> <b>and</b> divide by 4</p> <p>(c) (i) <math>x = 5</math> ruled</p> <p><math>y = 11</math> ruled</p> <p><math>x + y = 20</math> ruled</p> <p><math>x + 2y = 40</math> ruled</p> <p>Correct shading of <b>unwanted</b> region</p> <p>(ii) 29</p>	<p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>1</p> <p>2</p> <p>2</p> <p>1dep</p> <p>2</p>	<p>–1 once for strict inequalities in (i) to (iii)</p> <p>If there is a final inequality it must be the given one</p> <p>Must be on correct grid line</p> <p>Must be on correct grid line</p> <p><b>B1</b> for one axis intercept correct when extended if necessary but not parallel to an axis</p> <p><b>B1</b> for one axis intercept correct when extended if necessary but not parallel to an axis</p> <p>Dependent on 6 marks earned for the boundaries</p> <p><b>M1</b> for <math>x + y</math> evaluated where <math>(x, y)</math> is a point in their <b>quadrilateral and</b> <math>x</math> and <math>y</math> are integers</p>

8	9	<p>(a) <math>y = 2</math> oe  <math>y = 2x</math> oe  <math>y = -\frac{1}{2}x + 5</math> oe</p> <p>(b) <math>y \geq 2</math> oe  <math>y \leq 2x</math> oe  <math>y \leq -\frac{1}{2}x + 5</math> oe</p> <p>(c) (i) 4 [bushes], 3 [trees]    (ii) 2 [bushes], 4 [trees]    860</p>	<p>1  2  2  3  2  2  1</p>	<p><b>M1</b> for <math>y = kx</math>, <math>k \neq 0</math> or gradient 2 soi  <b>M1</b> for gradient <math>-\frac{1}{2}</math> soi or <math>y = kx + 5</math>oe  or <math>x + 2y = k</math> <math>k \neq 0</math> oe  If <math>L^2</math> <b>and</b> <math>L^3</math> both correct but interchanged then <b>SC3</b></p> <p><b>B1</b> for each correct inequality, allow in any order  After 0 scored, <b>SC1</b> for all inequalities reversed</p> <p><b>M1</b> for any correct trial using integer coordinates in region  or <math>30x + 200y = 720</math> seen</p> <p><b>M1</b> for any correct trial using integer coordinates in region</p>
9	3	<p>(a) <math>9 - 2x</math>, <math>7 - 2x</math> oe</p> <p>(b) <math>x(9 - 2x)(7 - 2x)</math>  <math>4x^3 - 32x^2 + 63x</math></p> <p>(c) 24 20</p> <p>(d) Correct curve</p> <p>(e) <math>0.65 \text{ to } 0.75 \leq x \leq 2</math> oe</p> <p>(f) (i) 36 to 37  (ii) 1.2 to 1.4</p>	<p>2  <b>M1FT</b>  <b>A1</b>  2  3  2  1  1</p>	<p><b>B1</b> for each, accept in any order</p> <p>Correct expansion and simplification with no errors</p> <p><b>B1</b> for each correct value</p> <p><b>B2FT</b> for 5 correct plots  or  <b>B1FT</b> for 3 or 4 correct plots</p> <p><b>B1</b> for 0.65 to 0.75 seen</p>

10	8	(a)	(i) $x \geq 100$ final answer	1	
			(ii) $y \geq 120$ final answer	1	
			(iii) $x + y \leq 300$ final answer	1	
			(iv) $40x + 80y \geq 16000$ or $0.4x + 0.8y \geq 160$	M1	with no errors seen but isw substitution of values after correct inequality
		(b)	$x = 100$ ruled	B1	
			$y = 120$ ruled	B1	
			$x + y = 300$ ruled	B1	
			$x + 2y = 400$ ruled	B2	Allow B1 for line with negative gradient passing through (400, 0) or (0, 200) when extended
		(c)	Correct shading	B1	Dep on all previous marks earned Condone any clear indication of the required region
			200	2	M1 for $x = 100$ and $y = 200$ selected or for $x \times 0.4 + y \times 0.8$ oe evaluated where $(x, y)$ is an integer point in <i>their</i> [unshaded] region
11	4	(a)	(i) $x \geq 5$ oe $y \leq 8$ oe $x + y \leq 15$ oe $y > x$ oe or $y \geq x + 1$	4	Condone $5 \leq x \leq 15$ Condone $0 < y \leq 8$  B1 for each – 1 for first occurrence of strict inequalities used in first 3 inequalities
			(ii) $x = 5$ ruled $y = 8$ ruled $x + y = 15$ ruled $y = x$ ruled broken line	1 1 1 1	Allow $y = x + 1$ ruled only after $y \geq x + 1$ in (a)(i)
			Correct region indicated	1dep	Dependent on all marks for lines earned Accept R written in correct quadrilateral or any other unambiguous indication or accept in triangle if $y = x + 1$ used and all marks for lines earned
		(b)	78	2	B1 for (7, 8) chosen or M1 for a calculation shown of the form $6x + 4.5y$ where $(x, y)$ is clearly in <i>their</i> region and both $x$ and $y$ are integers



12	10 (a)	$4x + 10y < 80$	1	With no errors seen
	(b)	$y > x$ $y \leq 6$ or $y < 7$	1	
	(c)	ruled broken line through (5, 6) to (10, 4)  ruled broken line $y = x$ ruled solid line $y = 6$ or broken $y = 7$  correct region indicated	B2  B1 B1 B1	Accept $0 \leq y \leq 6$ or $0 < y \leq 6$ or $0 \leq y < 7$ or $0 < y < 7$  SC1 for correct only at (5, 6) or (10, 4)  Must be consistent with <i>their</i> (b)
	(d)	76	2	SC1 for (4, 6) indicated or  $4x + 10y$ evaluated for (x, y) in <i>their</i> region, x, y integers
13	9(a)	$y = -2x + 5$ oe	3	B2 for $-2x + 5$ or M1 for gradient $= -1 \div \frac{1}{2}$ or better M1 for substituting (1, 3) into $y = (\text{their } m)x + c$ oe If 0 scored SC1 for (1, 3) satisfying their wrong equation ( $c \neq 0$ ) with gradient $\neq \frac{1}{2}$
	9(b)(i)	$x \geq 2$ oe $y \leq 5$ oe  $y \geq \frac{1}{2}x$ oe	4	SC3 for $x > 2$ and $y < 5$ and $y > \frac{1}{2}x$ OR B1 for $x \geq 2$ B1 for $y \leq 5$ B2 for $y \geq \frac{1}{2}x$ or M1 for $y \geq kx$ ( $k > 0$ ) OR SC2 for all three boundary lines identified but with incorrect sign(s)  If 0 scored SC1 for one or two correct boundary lines with incorrect sign(s)
	9(b)(ii)	(5, 4)	2	M1 for one trial of an integer point inside region or for $3x + 5y = 35$ drawn