

Matrices

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- 21 (a) \mathbf{A} is a (2×4) matrix, \mathbf{B} is a (3×2) matrix and \mathbf{C} is a (1×3) matrix.

Which two of the following matrix products is it possible to work out?

$$\mathbf{A}^2 \quad \mathbf{B}^2 \quad \mathbf{C}^2 \quad \mathbf{AB} \quad \mathbf{AC} \quad \mathbf{BA} \quad \mathbf{BC} \quad \mathbf{CA} \quad \mathbf{CB}$$

Answer(a) and [2]

- (b) Find the inverse of $\begin{pmatrix} \frac{1}{2} & \frac{3}{4} \\ \frac{1}{8} & \frac{1}{4} \end{pmatrix}$.

Simplify your answer as far as possible.

Answer(b) $\left(\begin{array}{c} \text{ } \\ \text{ } \\ \text{ } \end{array} \right)$ [3]

- (c) Explain why the matrix $\begin{pmatrix} 4 & 2 \\ 6 & 3 \end{pmatrix}$ does not have an inverse.

Answer(c) [1]

2) June 2010 V2

13

$$\mathbf{M} = \begin{pmatrix} 6 & -3 \\ 4 & 5 \end{pmatrix} \begin{pmatrix} x \\ 1 \end{pmatrix}$$

(a) Find the matrix \mathbf{M}

Answer(a) $\mathbf{M} =$

[2]

(b) Simplify $(x - 1)\mathbf{M}$.

Answer(b)

[2]

3) June 2010 V3

23

$$\mathbf{A} = \begin{pmatrix} 1 & 4 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$$

Find

(a) \mathbf{AB} ,

Answer(a) $\mathbf{AB} =$

[2]

(b) the inverse matrix \mathbf{B}^{-1} ,

Answer(b) $\mathbf{B}^{-1} =$

[2]

(c) \mathbf{BB}^{-1}

Answer(c) $\mathbf{BB}^{-1} =$

[1]

17

$$\mathbf{A} = \begin{pmatrix} 2 & 2 \\ 2 & -2 \end{pmatrix}$$

Work out

(a) \mathbf{A}^2 ,

Answer(a)

[2]

(b) \mathbf{A}^{-1} , the inverse of \mathbf{A} .

Answer(b)

$$\left(\quad \quad \right) [2]$$

18

$$\mathbf{A} = \begin{pmatrix} 2 & 4 \\ 5 & 3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 3 & -4 \\ -5 & 2 \end{pmatrix}$$

- (a) Work out \mathbf{AB} .

Answer(a)

[2]

- (b) Find $|\mathbf{B}|$, the determinant of \mathbf{B} .

Answer(b)

[1]

- (c) \mathbf{I} is the (2×2) identity matrix.
Find the matrix \mathbf{C} , where $\mathbf{C} = \mathbf{A} - 7\mathbf{I}$.

Answer(c)

[2]

21 (a)

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 6 \\ -4 \end{pmatrix}$$

(i) Work out \mathbf{AB} .

Answer(a)(i)

[2]

(ii) Work out \mathbf{BA} .

Answer(a)(ii)

[2]

(b) $\mathbf{C} = \begin{pmatrix} 3 & 1 \\ 1 & 1 \end{pmatrix}$

Find \mathbf{C}^{-1} , the inverse of \mathbf{C} .

Answer(b)

[2]

11 Work out.

(a) $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}^2$

Answer(a)

$$\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} [2]$$

(b) $\begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix}^{-1}$

Answer(b)

$$\begin{pmatrix} \quad & \quad \end{pmatrix} [2]$$

$$7 \quad \begin{pmatrix} 0 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 8 \\ 25 \end{pmatrix}$$

Find the value of a and the value of b

Answer $a =$

$b =$ [3]

20 (a) $\mathbf{N} = \begin{pmatrix} 2 \\ 6 \end{pmatrix}$. The order of the matrix \mathbf{N} is 2×1 .

$\mathbf{P} = (1 \ 3)$. The order of the matrix \mathbf{P} is 1×2 .

(i) Write down the order of the matrix \mathbf{NP} .

Answer(a)(i) [1]

(ii) Calculate \mathbf{PN} .

Answer(a)(ii) [1]

(b) $\mathbf{M} = \begin{pmatrix} 2 & 3 \\ 2 & 4 \end{pmatrix}$.

Find \mathbf{M}^{-1} , the inverse of \mathbf{M} .

Answer(b) $\mathbf{M}^{-1} =$ [2]

16

$$\mathbf{M} = \begin{pmatrix} 5 & 2 \\ -3 & 4 \end{pmatrix}$$

$$\mathbf{N} = \begin{pmatrix} -1 & -2 \\ 2 & 6 \end{pmatrix}$$

Calculate

(a) \mathbf{MN} ,

Answer(a) $\mathbf{MN} =$

[2]

(b) \mathbf{M}^{-1} , the inverse of \mathbf{M} .

Answer(b) $\mathbf{M}^{-1} =$

[2]

19 Find the values of x for which

(a) $\begin{pmatrix} 1 & 0 \\ 0 & 2x-7 \end{pmatrix}$ has no inverse,

Answer (a) $x = \dots$ [2]

(b) $\begin{pmatrix} 1 & 0 \\ 0 & x^2-8 \end{pmatrix}$ is the identity matrix,

Answer (b) $x = \dots$ or $x = \dots$ [3]

(c) $\begin{pmatrix} 1 & 0 \\ 0 & x-2 \end{pmatrix}$ represents a stretch with factor 3 and the x axis invariant.

Answer (c) $x = \dots$ [2]

17 $A = \begin{pmatrix} 2 & 4 \\ 1 & 3 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 2 \end{pmatrix}$

(a) Calculate BA .

Answer(a)

[2]

(b) Find A^{-1} , the inverse of A .

Answer(b)

[2]

19

$$\mathbf{M} = \begin{pmatrix} 5 & -4 \\ 2 & 3 \end{pmatrix}$$

Find

(a) \mathbf{M}^2 ,

Answer(a)

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

[2]

(b) $2\mathbf{M}$,

Answer(b)

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

[1]

(c) $|\mathbf{M}|$, the determinant of \mathbf{M} ,

Answer(c)

[1]

(d) \mathbf{M}^{-1} .

Answer(d)

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

[2]

22 (a) $M = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$

Find M^{-1} , the inverse of M.

Answer(a)

$$\begin{pmatrix} & \\ & \end{pmatrix}$$

[2]

- (b) D, E and X are 2×2 matrices.
I is the identity 2×2 matrix.

(i) Simplify DI.

Answer(b)(i)

[1]

(ii) $DX = E$

Write X in terms of D and E.

Answer(b)(ii) X =

[1]

24 $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 4 & 3 \\ 1 & 2 \end{pmatrix}$

Find

(a) \mathbf{AB} ,

Answer(a) $\mathbf{AB} =$ [2]

(b) \mathbf{B}^{-1} , the inverse of \mathbf{B}

Answer(b) $\mathbf{B}^{-1} =$ [2]

17 $M = \begin{pmatrix} 2 & 3 \\ 3 & 6 \end{pmatrix}$ $N = \begin{pmatrix} 2 & 1 & 5 \\ 1 & 7 & 2 \end{pmatrix}$

(a) Work out MN .

Answer(a)

[2]

(b) Find M^{-1} , the inverse of M .

Answer(b)

[2]

17

$$\mathbf{M} = \begin{pmatrix} 2 & 1 \\ 4 & 6 \end{pmatrix}$$

$$\mathbf{N} = \begin{pmatrix} 5 & 0 \\ 1 & 5 \end{pmatrix}$$

- (a) Work out \mathbf{MN} .

Answer(a) $\mathbf{MN} =$

[2]

- (b) Find \mathbf{M}^{-1} .

Answer(b) $\mathbf{M}^{-1} =$

[2]

$$11 \quad A = \begin{pmatrix} 3 & -1 \\ 4 & 2 \end{pmatrix} \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Work out the following.

(a) AI

Answer(a) $AI =$

[1]

(b) A^{-1}

Answer(b) $A^{-1} =$

[2]

15 $\mathbf{M} = \begin{pmatrix} 4 & 2 \\ 3 & 5 \end{pmatrix}$

Find

- (a) \mathbf{M}^2 ,

Answer(a)

[2]

- (b) the determinant of \mathbf{M} .

Answer(b) [1]

18

$$\mathbf{A} = \begin{pmatrix} 5 & 2 \\ 4 & 3 \end{pmatrix}$$

- (a) Calculate \mathbf{A}^2 .

Answer(a)

[2]

- (b) Calculate \mathbf{A}^{-1} , the inverse of \mathbf{A} .

Answer(b)

[2]

21) November 2014 V1

14 $\mathbf{A} = \begin{pmatrix} 2 & 8 \\ 1 & 4 \end{pmatrix}$

Work out $\mathbf{A}^2 - 4\mathbf{A}$

Answer

()

[3]

22) November 2014 V2

11 $\mathbf{A} = \begin{pmatrix} 3 & -2 \\ 1 & 4 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ 5 & 7 \end{pmatrix}$

(a) Calculate \mathbf{BA}

Answer(a) $\mathbf{BA} =$

[2]

(b) Find the determinant of \mathbf{A} .

www.Q8Maths.com Answer(b) [1]

22 (a) Calculate $\begin{pmatrix} 3 & 7 \\ -1 & 4 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ 4 & 2 \end{pmatrix}$.

Answer(a)

[2]

(b) Calculate the inverse of $\begin{pmatrix} 5 & 3 \\ 6 & 4 \end{pmatrix}$.

Answer(b)

$$\left(\quad \right)$$

[2]

24) June 2015 V2

11 $\mathbf{M} = \begin{pmatrix} 3 & 1 \\ 11 & 2 \end{pmatrix}$

Find \mathbf{M}^{-1} , the inverse of \mathbf{M} .

Answer

$$\left(\quad \quad \right)$$

[2]

25) November 2015 V1

18 (a) Work out $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 5 & 3 \\ 2 & 1 \end{pmatrix}$.

Answer(a)

$$\left(\quad \quad \right)$$

[2]

(b) Find the inverse of $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

Answer(b)

$$\left(\quad \quad \right)$$

[2]

(c) Explain why it is not possible to work out $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Answer(c) [1]

26) November 2015 V2

7

$$\mathbf{M} = \begin{pmatrix} 3 & -4 \\ 2 & 4 \end{pmatrix}$$

$$\mathbf{N} = \begin{pmatrix} 5 & 0 \\ 1 & 2 \end{pmatrix}$$

Calculate \mathbf{MN} .

Answer $\begin{pmatrix} \quad & \quad \end{pmatrix}$ [2]

27) November 2015 V3

13

$$\mathbf{M} = \begin{pmatrix} 7 & u \\ 2 & 3 \end{pmatrix} \text{ and } |\mathbf{M}| = 1.$$

Find the value of u .

Answer $u = \dots$ [2]

15 $A = \begin{pmatrix} 8 & 3 \\ 4 & 2 \end{pmatrix}$

Find

(a) A^2 ,

Answer(a) $A^2 = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$ [2]

(b) A^{-1} .

Answer(b) $A^{-1} = \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$ [2]

29) March 2016 V2

8 Find the inverse of the matrix $\begin{pmatrix} 3 & 2 \\ -8 & 7 \end{pmatrix}$

$\left(\quad \right)$ [2]

30) June 2016 V2

22 $M = \begin{pmatrix} 5 & 1 \\ 3 & 2 \end{pmatrix}$

(a) Work out $4M$.

$\left(\quad \right)$ [1]

(b) Work out M^2 .

$\left(\quad \right)$ [2]

(c) Find M^{-1} , the inverse of M .

$\left(\quad \right)$ [2]

31) November 2016 V1

15 Work out.

(a) $2 \begin{pmatrix} 3 \\ 5 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ [1]

(b) $(1 \ 2) \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ [2]

32) November 2016 V1

16 $\vec{BC} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $\vec{BA} = \begin{pmatrix} 5 \\ 6 \end{pmatrix}$

(a) Find \vec{CA} .

$$\vec{CA} = \left(\quad \right) [2]$$

(b) Work out \vec{BA} .

[2]

19 (a) Find the inverse of $\begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}$

$$\left(\quad \quad \right) [2]$$

(b) The matrix $\begin{pmatrix} w & 9 \\ 4 & w & 12 \end{pmatrix}$ does not have an inverse.

Calculate the value of w .

$$w = \dots \dots \dots [4]$$

25 $\mathbf{A} = \begin{pmatrix} 4 & 2 \\ 2 & 1 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 7 & 3 \\ 4 & 5 \end{pmatrix}$ $\mathbf{C} = \begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & -1 \end{pmatrix}$ $\mathbf{D} = \begin{pmatrix} 9 \\ 0 \end{pmatrix}$

(a) Which of these four matrix calculations is **not** possible?

$\mathbf{A} + \mathbf{B}$

$3\mathbf{C}$

$\mathbf{C}\mathbf{B}$

$\mathbf{A}\mathbf{D}$

[1]

(b) Calculate \mathbf{AB}

[2]

(c) Work out \mathbf{B}^{-1} , the inverse of \mathbf{B} .

$$\left(\quad \right)$$
 [2]

(d) Explain why matrix \mathbf{A} does not have an inverse.

[1]

20 $\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 9 & 9 \end{pmatrix}$

$\mathbf{B} = \begin{pmatrix} 0 & 1 \\ 9 & 8 \end{pmatrix}$

$\mathbf{C} = \begin{pmatrix} 1 & 1 \\ 3 & 3 \end{pmatrix}$

$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(a) Here are four matrix calculations.

\mathbf{AI}

\mathbf{IA}

\mathbf{C}^2

$\mathbf{B} + \mathbf{I}$

Work out which matrix calculation does **not** give the answer $\begin{pmatrix} 1 & 1 \\ 9 & 9 \end{pmatrix}$.

[2]

(b) Find $|\mathbf{B}|$.

[1]

(c) Explain why matrix \mathbf{A} has no inverse.

[1]