Functions & Graphs
2002 - 2011

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17 $f(x) = \dfrac{1}{x+4} \quad (x \neq -4)$

$g(x) = x^2 - 3x$

$h(x) = x^3 + 1$

(a) Work out fg(1).

Answer(a) ..... [2]

(b) Find $h^{-1}(x)$.

Answer(b) $h^{-1}(x) =$ ..... [2]

(c) Solve the equation $g(x) = -2$.

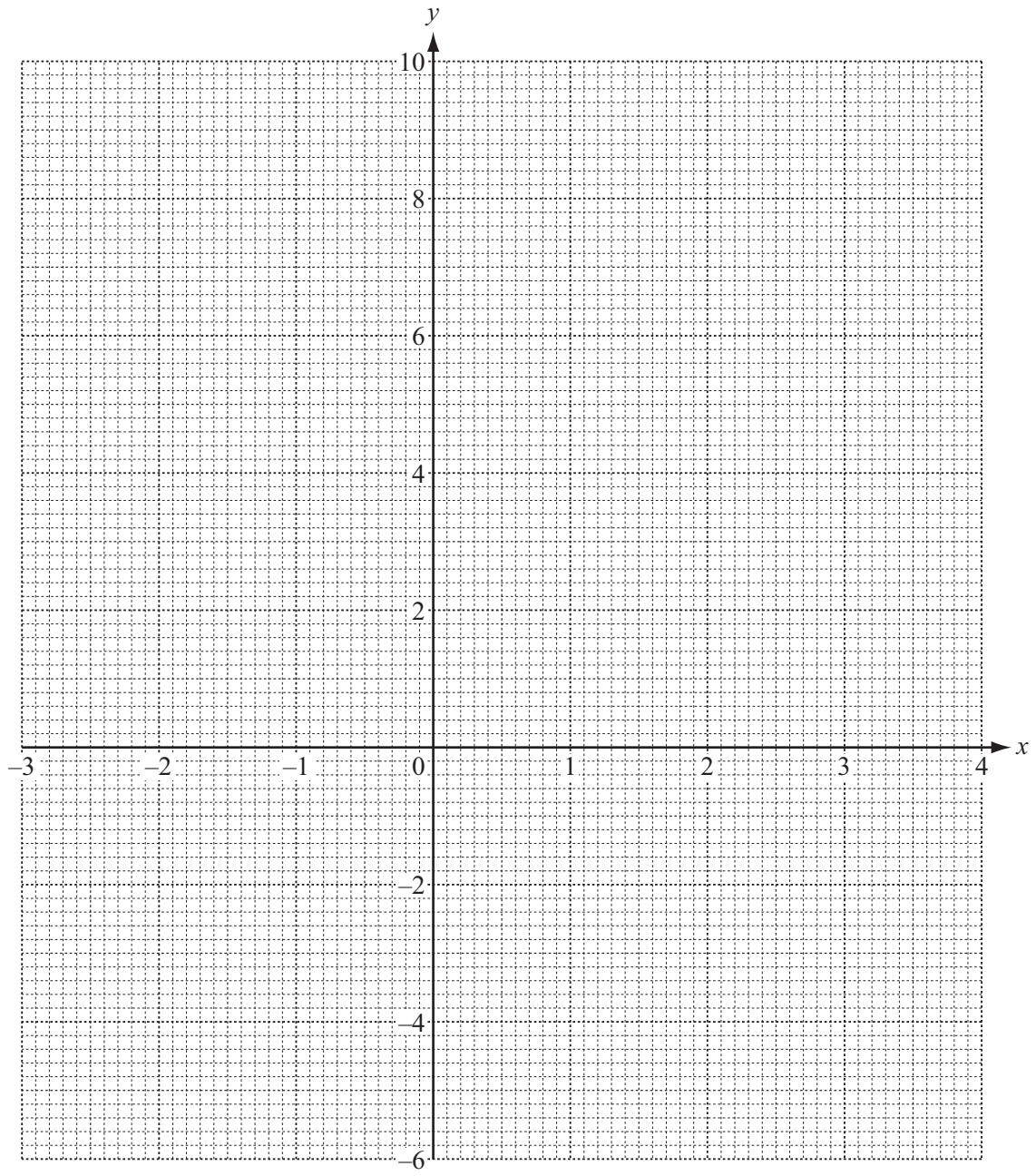
Answer(c) $x =$ ..... or $x =$ ..... [3]

5 **(a)** Complete the table for the function $f(x) = \frac{x^3}{2} - 3x - 1$.

x	–3	–2	–1.5	–1	0	1	1.5	2	3	3.5
f(x)	–5.5		1.8	1.5		–3.5	–3.8	–3		9.9

[3]

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 3.5$.



[4]

(c) Use your graph to

(i) solve f(x) = 0.5,

Answer(c)(i) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(ii) find the inequalities for k, so that f(x) = k has only 1 answer.

Answer(c)(ii) $k <$ .....

$k >$ ..... [2]

(d) (i) On the same grid, draw the graph of $y = 3x - 2$ for $-1 \leqslant x \leqslant 3.5$. [3]

(ii) The equation $\frac{x^3}{2} - 3x - 1 = 3x - 2$ can be written in the form $x^3 + ax + b = 0$.
Find the values of a and b.

Answer(d)(ii) $a =$ ..... and $b =$ ..... [2]

(iii) Use your graph to find the **positive** answers to $\frac{x^3}{2} - 3x - 1 = 3x - 2$ for $-3 \leqslant x \leqslant 3.5$.

Answer(d)(iii) $x =$ ..... or $x =$ ..... [2]

2

$$f(x) = 4x - 2$$

$$g(x) = \frac{2}{x} + 1$$

$$h(x) = x^2 + 3$$

(a) (i) Find the value of hf(2).

Answer(a)(i) ..... [2]

(ii) Write fg(x) in its simplest form.

Answer(a)(ii) fg(x) = ... [2]

(b) Solve g(x) = 0.2.

Answer(b) x = ... [2]

(c) Find the value of gg(3).

Answer(c) ..... [2]

(d) (i) Show that f(x) = g(x) can be written as $4x^2 - 3x - 2 = 0$.

Answer (d)(i)

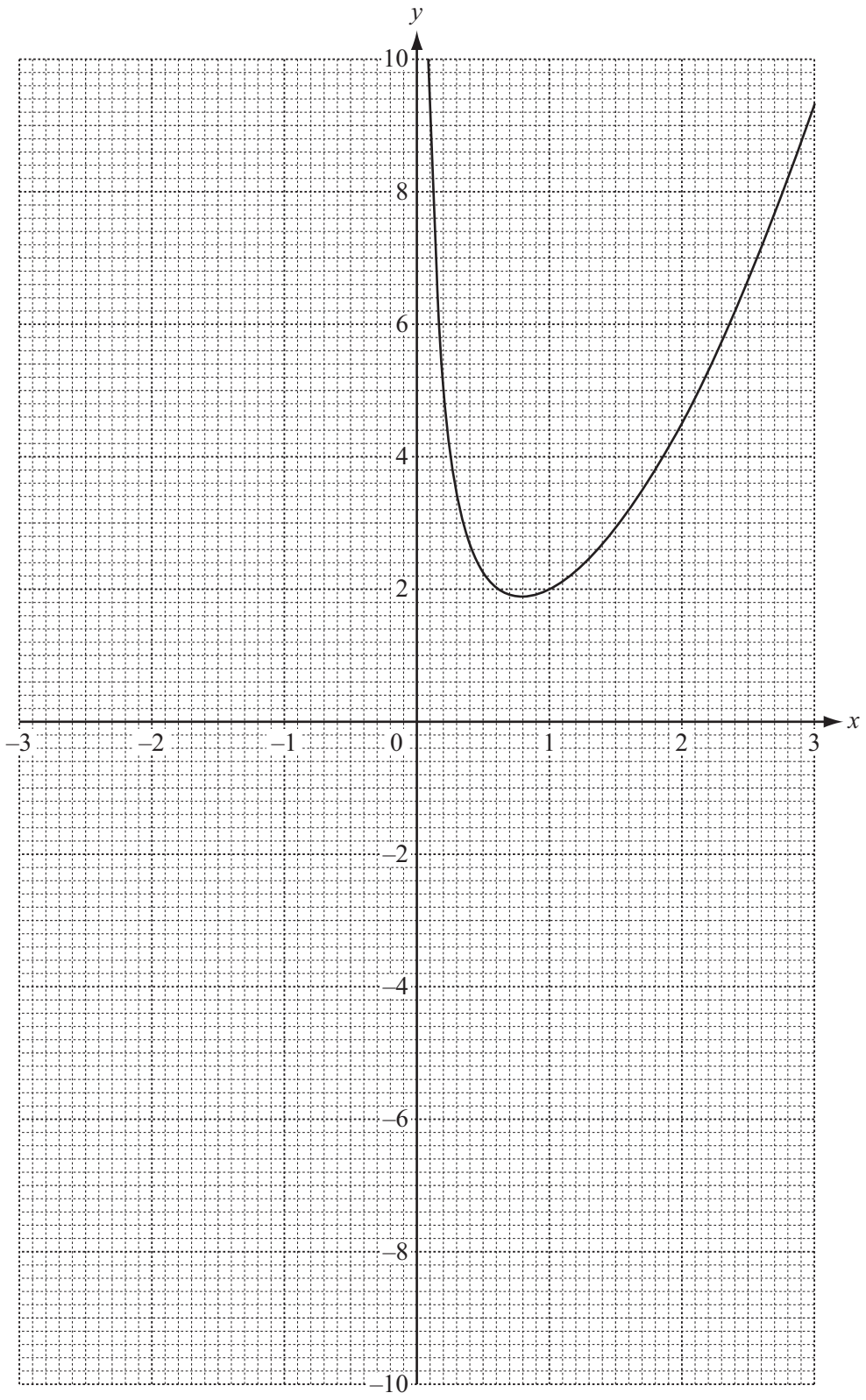
[1]

(ii) Solve the equation $4x^2 - 3x - 2 = 0$.

Show all your working and give your answers correct to 2 decimal places.

Answer(d)(ii) x = ... or x = ... [4]

7 The diagram shows the accurate graph of $y = f(x)$ where $f(x) = \frac{1}{x} + x^2$ for $0 < x \leqslant 3$.



(a) Complete the table for $f(x) = \frac{1}{x} + x^2$.

x	-3	-2	-1	-0.5	-0.3	-0.1
$f(x)$		3.5	0	-1.8		

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x < 0$. [3]

(c) By drawing a tangent, work out an estimate of the gradient of the graph where $x = 2$.

Answer(c) ..... [3]

(d) Write down the inequality satisfied by k when $f(x) = k$ has three answers.

Answer(d) ..... [1]

(e) (i) Draw the line $y = 1 - x$ on the grid for $-3 \leqslant x \leqslant 3$. [2]

(ii) Use your graphs to solve the equation $1 - x = \frac{1}{x} + x^2$.

Answer(e)(ii) $x =$ ..... [1]

(f) (i) Rearrange $x^3 - x^2 - 2x + 1 = 0$ into the form $\frac{1}{x} + x^2 = ax + b$, where a and b are integers.

Answer(f)(i)

[2]

(ii) Write down the equation of the line that could be drawn on the graph to solve $x^3 - x^2 - 2x + 1 = 0$.

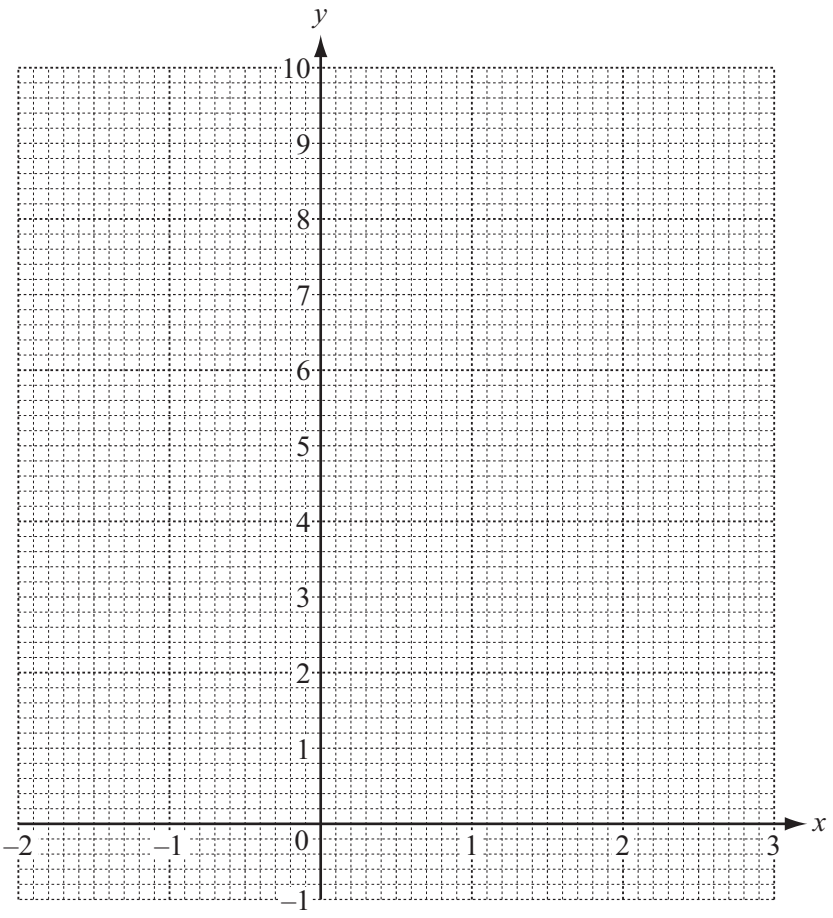
Answer(f)(ii) $y =$ ..... [1]

2 **(a)** Complete the table of values for $y = 2^x$.

x	–2	–1	0	1	2	3
y	0.25		1	2		8

[2]

(b) On the grid, draw the graph of $y = 2^x$ for $-2 \leqslant x \leqslant 3$.



[3]

(c) (i) On the grid, draw the straight line which passes through the points (0, 2) and (3, 8). [1]

(ii) The equation of this line is $y = mx + 2$.

Show that the value of m is 2.

Answer(c)(ii)

[1]

(iii) One answer to the equation $2^x = 2x + 2$ is $x = 3$.

Use your graph to find the other answer.

Answer(c)(iii) $x =$ ..... [1]

(d) Draw the tangent to the curve at the point where $x = 1$.

Use this tangent to calculate an estimate of the gradient of $y = 2^x$ when $x = 1$.

Answer(d) ..... [3]

8 $f(x) = x^2 + x - 1$ $g(x) = 1 - 2x$ $h(x) = 3^x$

(a) Find the value of $hg(-2)$.

Answer(a) ..... [2]

(b) Find $g^{-1}(x)$.

Answer(b) $g^{-1}(x) =$ ..... [2]

(c) Solve the equation $f(x) = 0$.
Show all your working and give your answers correct to 2 decimal places.

Answer(c) $x =$ ..... or $x =$ ..... [4]

(d) Find $fg(x)$.
Give your answer in its simplest form.

Answer(d) $fg(x) =$ ..... [3]

(e) Solve the equation $h^{-1}(x) = 2$.

Answer(e) $x =$ ..... [1]

(b) Using a scale of 2 cm to represent 1 minute on the horizontal t-axis and 2 cm to represent 10 metres on the vertical d-axis, draw the graph of $d = (t + 1)^2 + \frac{48}{(t + 1)} - 20$ for $0 \leqslant t \leqslant 7$. [6]

(c) Mark and label F the point on your graph when the fish is 12 metres from Dimitra and swimming **away** from her. Write down the value of t at this point, correct to one decimal place. [2]

(d) For how many minutes is the fish less than 10 metres from Dimitra? [2]

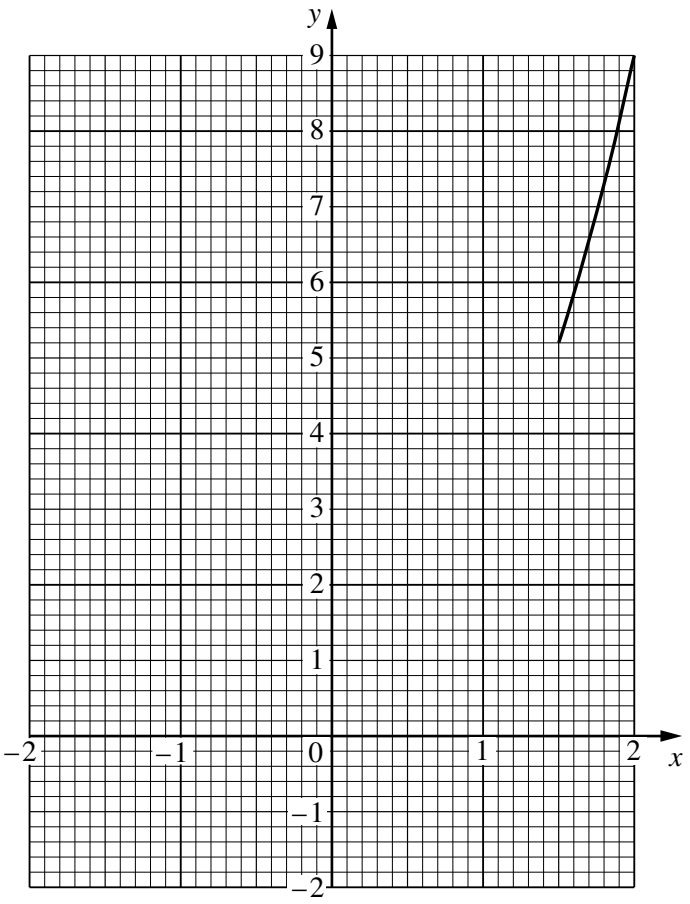
(e) By drawing a suitable line on your grid, calculate the speed of the fish when $t = 2.5$. [4]

20 **(a)** Complete the table of values for $y = 3^x$.

x	−2	−1.5	−1	−0.5	0	0.5	1	1.5	2
y		0.2						5.2	9

[3]

(b) Use your table to complete the graph of $y = 3^x$ for $-2 \leqslant x \leqslant 2$.



[2]

(c) Use the graph to find the solution of the equation

$$3^x = 6.$$

Answer (c) $x =$ ..... [1]

4 Answer the whole of this question on a sheet of graph paper.

x	−4	−3	−2	−1	0	1	2	3	4
f(x)	−8	4.5	8	5.5	0	−5.5	−8	−4.5	8

(a) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 4 units on the y-axis, draw axes for $-4 \leqslant x \leqslant 4$ and $-8 \leqslant y \leqslant 8$.
Draw the curve $y = f(x)$ using the table of values given above. [5]

(b) Use your graph to solve the equation $f(x) = 0$. [2]

(c) On the same grid, draw $y = g(x)$ for $-4 \leqslant x \leqslant 4$, where $g(x) = x + 1$. [2]

(d) Write down the value of

(i) g(1),

(ii) fg(1),

(iii) $g^{-1}(4)$,

(iv) the **positive** solution of $f(x) = g(x)$. [4]

(e) Draw the tangent to $y = f(x)$ at $x = 3$. Use it to calculate an estimate of the gradient of the curve at this point. [3]

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5 (a) Calculate the area of an equilateral triangle with sides 10 cm. [2]

(b) Calculate the radius of a circle with circumference 10 cm. [2]

(c)

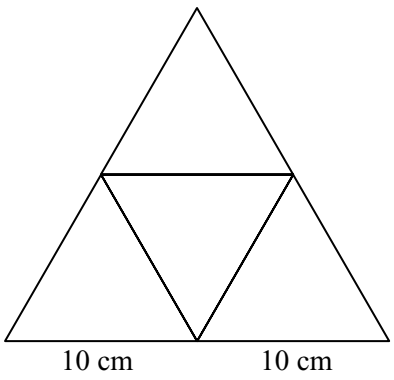

Diagram 1

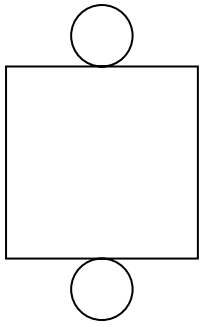
Diagram 2

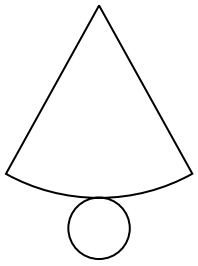
Diagram 3

The diagrams represent the nets of 3 solids. Each straight line is 10 cm long. Each circle has circumference 10 cm. The arc length in Diagram 3 is 10 cm.

(i) Name the solid whose net is Diagram 1. Calculate its surface area. [3]

(ii) Name the solid whose net is Diagram 2. Calculate its volume. [4]

(iii) Name the solid whose net is Diagram 3. Calculate its perpendicular height. [4]

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19 $f(x) = \frac{x+1}{2}$ and $g(x) = 2x + 1$.

(a) Find the value of $gf(9)$.

Answer(a) ..... [1]

(b) Find $gf(x)$, giving your answer in its simplest form.

Answer(b) ..... [2]

(c) Solve the equation $g^{-1}(x) = 1$.

Answer(c) ..... [2]

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20 **(a)** Factorise completely $12x^2 - 3y^2$.

Answer(a) ..... [2]

(b) **(i)** Expand $(x-3)^2$.

Answer(b)(i) ..... [2]

(ii) $x^2 - 6x + 10$ is to be written in the form $(x-p)^2 + q$.
Find the values of p and q.

Answer(b)(ii) p = ... q = ... [2]

---

2 Answer all of this question on a sheet of graph paper.

(a) $f(x) = x^2 - x - 3$.

x	-3	-2	-1	0	1	2	3	4
f(x)	p	3	-1	-3	q	-1	3	r

(i) Find the values of p, q and r. [3]

(ii) Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant 4$.
Use a scale of 1 cm to represent 1 unit on each axis. [4]

(iii) By drawing a suitable line, estimate the gradient of the graph at the point where $x = -1$. [3]

(b) $g(x) = 6 - \frac{x^3}{3}$.

x	-2	-1	0	1	2	3
g(x)	8.67	u	v	5.67	3.33	-3

(i) Find the values of u and v. [2]

(ii) On the same grid as **part (a) (ii)** draw the graph of $y = g(x)$ for $-2 \leqslant x \leqslant 3$. [4]

(c) (i) Show that the equation f(x) = g(x) simplifies to $x^3 + 3x^2 - 3x - 27 = 0$. [1]

(ii) Use your graph to write down a solution of the equation $x^3 + 3x^2 - 3x - 27 = 0$. [1]

4 **Answer the whole of this question on a sheet of graph paper.**

The table gives values of $f(x) = 2^x$, for $-2 \leqslant x \leqslant 4$.

x	-2	-1	0	1	2	3	4
f(x)	p	0.5	q	2	4	r	16

(a) Find the values of p, q and r. [3]

(b) Using a scale of 2 cm to 1 unit on the x-axis and 1 cm to 1 unit on the y-axis, draw the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 4$. [5]

(c) Use your graph to solve the equation $2^x = 7$. [1]

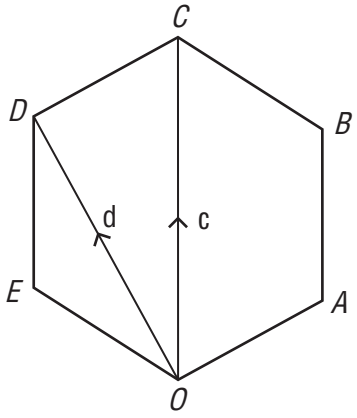
(d) What value does f(x) approach as x decreases? [1]

(e) By drawing a tangent, estimate the gradient of the graph of $y = f(x)$ when $x = 1.5$. [3]

(f) On the same grid draw the graph of $y = 2x + 1$ for $0 \leqslant x \leqslant 4$. [2]

(g) Use your graph to find the non-integer solution of $2^x = 2x + 1$. [2]

5



NOT TO SCALE

OABCDE is a regular hexagon.
With O as origin the position vector of C is **c** and the position vector of D is **d**.

(a) Find, in terms of **c** and **d**,

(i) $\overrightarrow{DC}$, [1]

(ii) $\overrightarrow{OE}$, [2]

(iii) the position vector of B. [2]

(b) The sides of the hexagon are each of length 8 cm.

Calculate

(i) the size of angle ABC, [1]

(ii) the area of triangle ABC, [2]

(iii) the length of the straight line AC, [3]

(iv) the area of the hexagon. [3]

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16 The function f(x) is given by

$$f(x) = 3x - 1.$$

Find, in its simplest form,

(a) $f^{-1}f(x)$,

Answer(a) ..... [1]

(b) $ff(x)$.

Answer(b) ..... [2]

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17 **(a)** $\sqrt{32} = 2^p$. Find the value of p.

Answer(a) $p =$ ..... [2]

(b) $\sqrt[3]{\frac{1}{8}} = 2^q$. Find the value of q.

Answer(b) $q =$ ..... [2]

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18 The equation of a straight line can be written in the form $3x + 2y - 8 = 0$.

(a) Rearrange this equation to make y the subject.

Answer(a) $y =$ ..... [2]

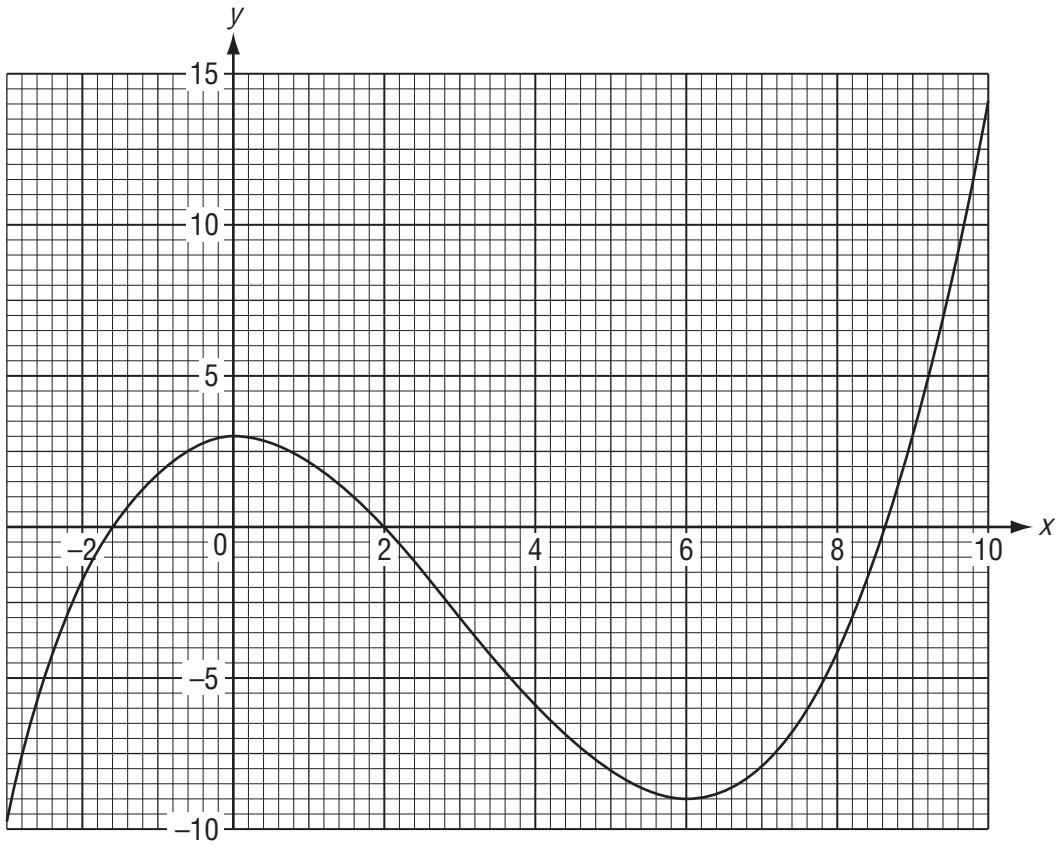
(b) Write down the gradient of the line.

Answer(b) ..... [1]

(c) Write down the co-ordinates of the point where the line crosses the y axis.

Answer(c) (..... , ...) [1]

4



The diagram shows the accurate graph of $y = f(x)$.

(a) Use the graph to find

(i) f(0), [1]

(ii) f(8). [1]

(b) Use the graph to solve

(i) $f(x) = 0$, [2]

(ii) $f(x) = 5$. [1]

(c) k is an integer for which the equation $f(x) = k$ has exactly two solutions.

Use the graph to find the two values of k. [2]

(d) Write down the range of values of x for which the graph of $y = f(x)$ has a negative gradient. [2]

(e) The equation $f(x) + x - 1 = 0$ can be solved by drawing a line on the grid.

(i) Write down the equation of this line. [1]

(ii) How many solutions are there for $f(x) + x - 1 = 0$? [1]

8 Answer the whole of this question on a sheet of graph paper.
Use one side for your working and one side for your graphs.

Alaric invests \$100 at 4% per year **compound interest.**

(a) How many dollars will Alaric have after 2 years? [2]

(b) After x years, Alaric will have y dollars.
He knows a formula to calculate y.
The formula is $y = 100 \times 1.04^x$

x (Years)	0	10	20	30	40
y (Dollars)	100	p	219	q	480

Use this formula to calculate the values of p and q in the table. [2]

(c) Using a scale of 2 cm to represent 5 years on the x-axis and 2 cm to represent \$50 on the y-axis, draw an x-axis for $0 \leqslant x \leqslant 40$ and a y-axis for $0 \leqslant y \leqslant 500$.

Plot the five points in the table and draw a smooth curve through them. [5]

(d) Use your graph to estimate

(i) how many dollars Alaric will have after 25 years, [1]

(ii) how many years, to the nearest year, it takes for Alaric to have \$200. [1]

(e) Beatrice invests \$100 at 7% per year **simple interest**.

(i) Show that after 20 years Beatrice has \$240. [2]

(ii) How many dollars will Beatrice have after 40 years? [1]

(iii) On the **same grid**, draw a graph to show how the \$100 which Beatrice invests will increase during the 40 years. [2]

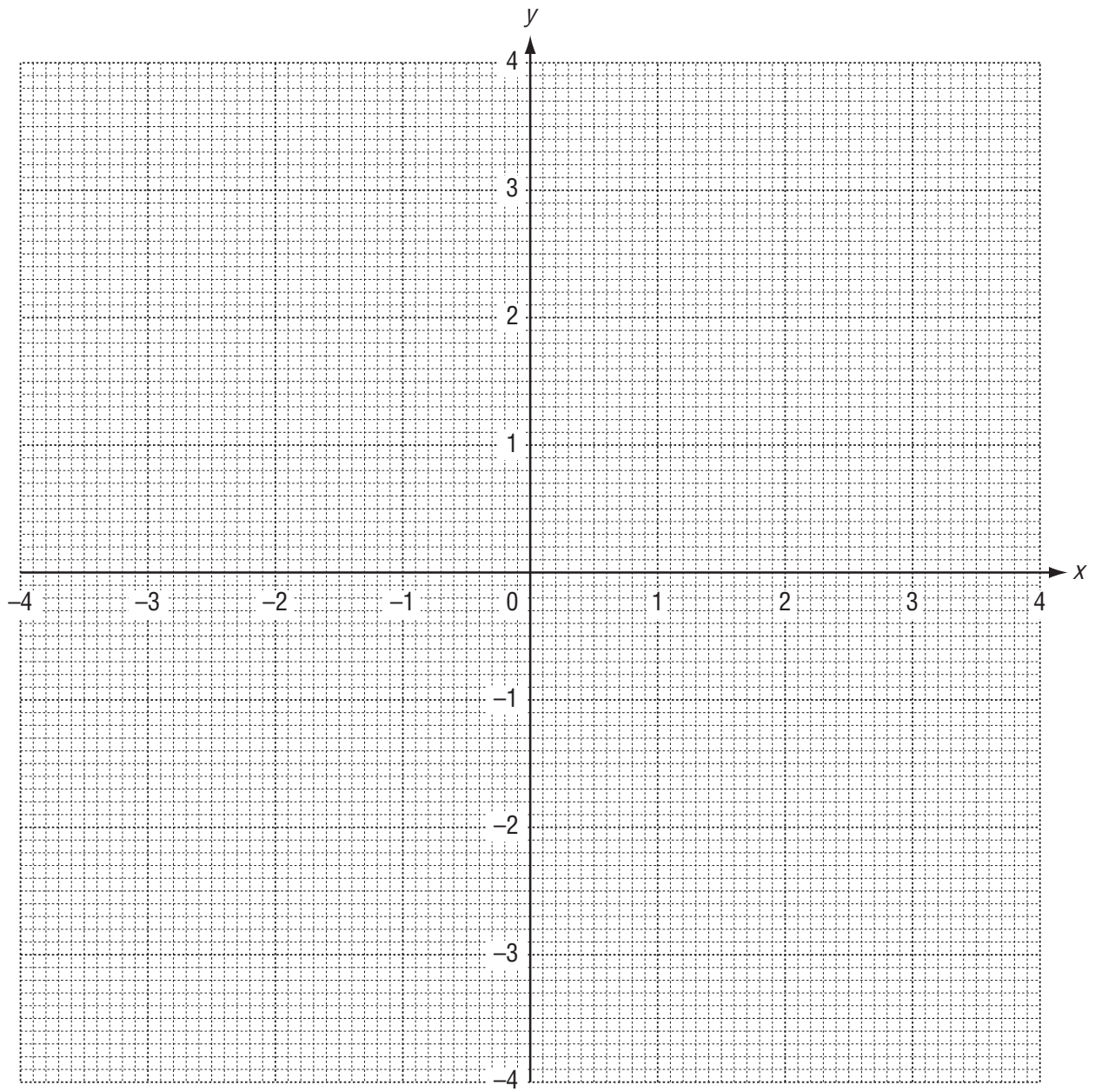
(f) Alaric first has more than Beatrice after n years.
Use your graphs to find the value of n. [1]

5 **(a)** The table shows some values for the equation $y = \frac{x}{2} - \frac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

x	–4	–3	–2	–1.5	–1	–0.5	0.5	1	1.5	2	3	4
y	–1.5	–0.83	0	0.58			–3.75		–0.58	0	0.83	1.5

(i) Write the missing values of y in the empty spaces. [3]

(ii) On the grid, draw the graph of $y = \frac{x}{2} - \frac{2}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.



[5]

(b) Use your graph to solve the equation $\frac{x}{2}-\frac{2}{x}=1$.

Answer(b) $x =$ ..... or $x =$ ..... [2]

(c) **(i)** By drawing a tangent, work out the gradient of the graph where $x = 2$.

Answer(c)(i) ..... [3]

(ii) Write down the gradient of the graph where $x = -2$.

Answer(c)(ii) ..... [1]

(d) **(i)** On the grid, draw the line $y = -x$ for $-4 \leqslant x \leqslant 4$. [1]

(ii) Use your graphs to solve the equation $\frac{x}{2}-\frac{2}{x}=-x$.

Answer(d)(ii) $x =$ ..... or $x =$ ..... [2]

(e) Write down the equation of a straight line which passes through the origin and does **not** intersect the graph of $y=\frac{x}{2}-\frac{2}{x}$.

Answer(e) ..... [2]

10 $f(x) = 2x - 1$ $g(x) = x^2 + 1$ $h(x) = 2^x$

(a) Find the value of

(i) $f\left(-\frac{1}{2}\right)$,

Answer(a)(i) ..... [1]

(ii) $g(-5)$,

Answer(a)(ii) ..... [1]

(iii) $h(-3)$.

Answer(a)(iii) ..... [1]

(b) Find the inverse function $f^{-1}(x)$.

Answer(b) $f^{-1}(x) =$ ..... [2]

(c) $g(x) = z$.
Find x in terms of z.

Answer(c) $x =$ ..... [2]

(d) Find $gf(x)$, in its simplest form.

Answer(d) $gf(x) =$ ..... [2]

(e) h(x) = 512.
Find the value of x.

Answer(e) x = ... [1]

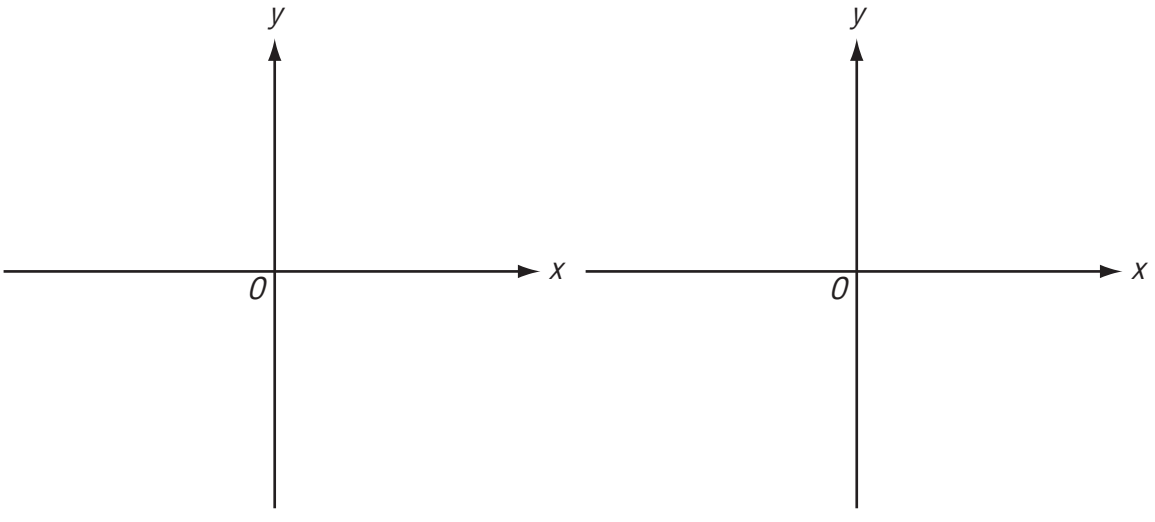
(f) Solve the equation $2f(x) + g(x) = 0$, giving your answers correct to 2 decimal places.

Answer(f) x = ... or x = ... [5]

(g) Sketch the graph of

(i) $y = f(x)$,

(ii) $y = g(x)$.



(i) $y = f(x)$ (ii) $y = g(x)$

[3]

20 $f(x) = (x - 1)^3$ $g(x) = (x - 1)^2$ $h(x) = 3x + 1$

(a) Work out fg(-1).

Answer(a) ..... [2]

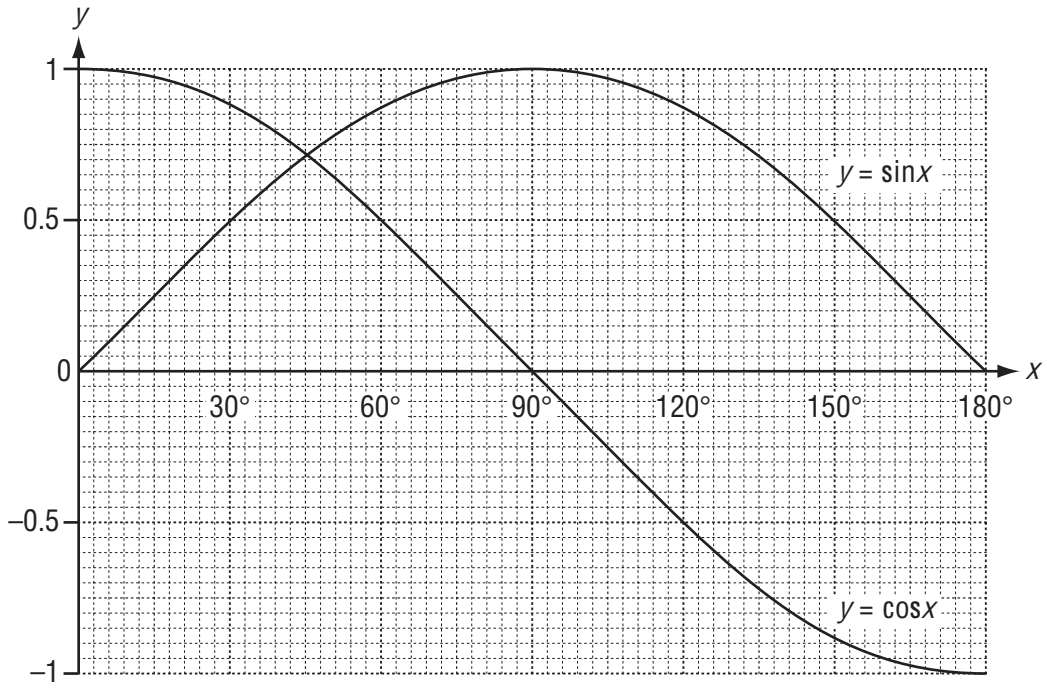
(b) Find $gh(x)$ in its simplest form.

Answer(b) ..... [2]

(c) Find $f^{-1}(x)$.

Answer(c) ..... [2]

8



The diagram shows accurate graphs of $y = \sin x$ and $y = \cos x$ for $0° \leqslant x \leqslant 180°$.

Use the graph to solve the equations

(a) $\sin x - \cos x = 0$,

Answer(a) $x =$ ..... [1]

(b) $\sin x - \cos x = 0.5$.

Answer(b) $x =$ ..... [2]

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9 A fence is made from 32 identical pieces of wood, each of length 2 metres correct to the nearest centimetre.

Calculate the lower bound for the total length of the wood used to make this fence.

Write down your full calculator display.

Answer ..... m [3]

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18 **(a)** $f(x) = 1 - 2x$.

(i) Find f(-5).

Answer(a)(i) ..... [1]

(ii) $g(x) = 3x - 2$.

Find $gf(x)$. Simplify your answer.

Answer(a)(ii) ..... [2]

(b) $h(x) = x^2 - 5x - 11$.

Solve $h(x) = 0$.

Show all your working and give your answer correct to 2 decimal places.

Answer(b) $x =$ ..... or $x =$ ..... [4]

19 The braking distance, d metres, for Alex's car travelling at v km/h is given by the formula

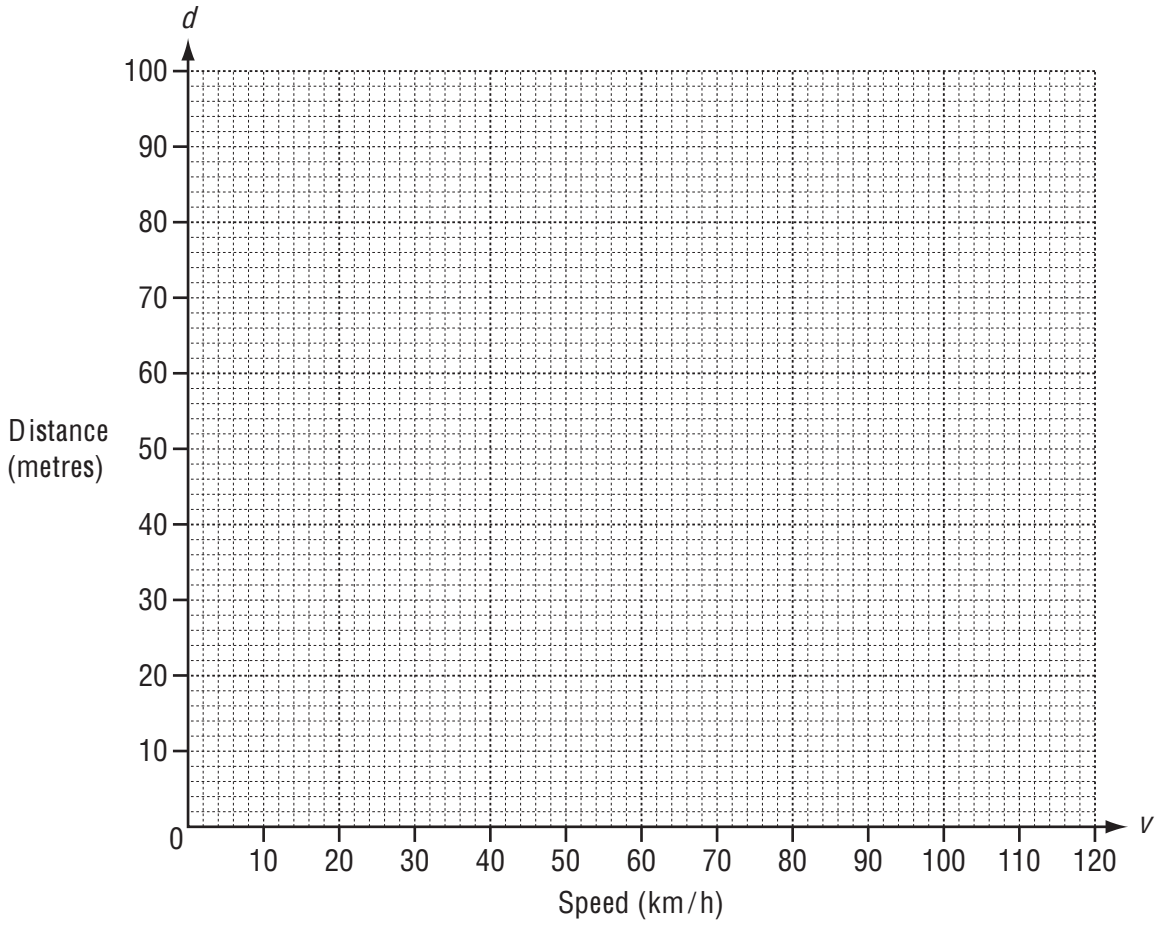
$$200d = v(v + 40).$$

(a) Calculate the missing values in the table.

v (km/h)	0	20	40	60	80	100	120
d (metres)	0		16		48		96

[2]

(b) On the grid below, draw the graph of $200d = v(v + 40)$ for $0 \leqslant v \leqslant 120$.



[3]

(c) Find the braking distance when the car is travelling at 110 km/h.

Answer(c) ..... m [1]

(d) Find the speed of the car when the braking distance is 80 m.

Answer(d) ..... km/h [1]

18 $f(x) = x^2 + 2$ $g(x) = (x + 2)^2$ $h(x) = 3x - 5$

Find

(a) $gf(-2)$,

Answer(a) ..... [2]

(b) $h^{-1}(22)$.

Answer(b) ..... [2]

8 **(a)** $f(x) = 2^x$

Complete the table.

x	–2	–1	0	1	2	3	4
$y = f(x)$		0.5	1	2	4		

[3]

(b) $g(x) = x(4 - x)$

Complete the table.

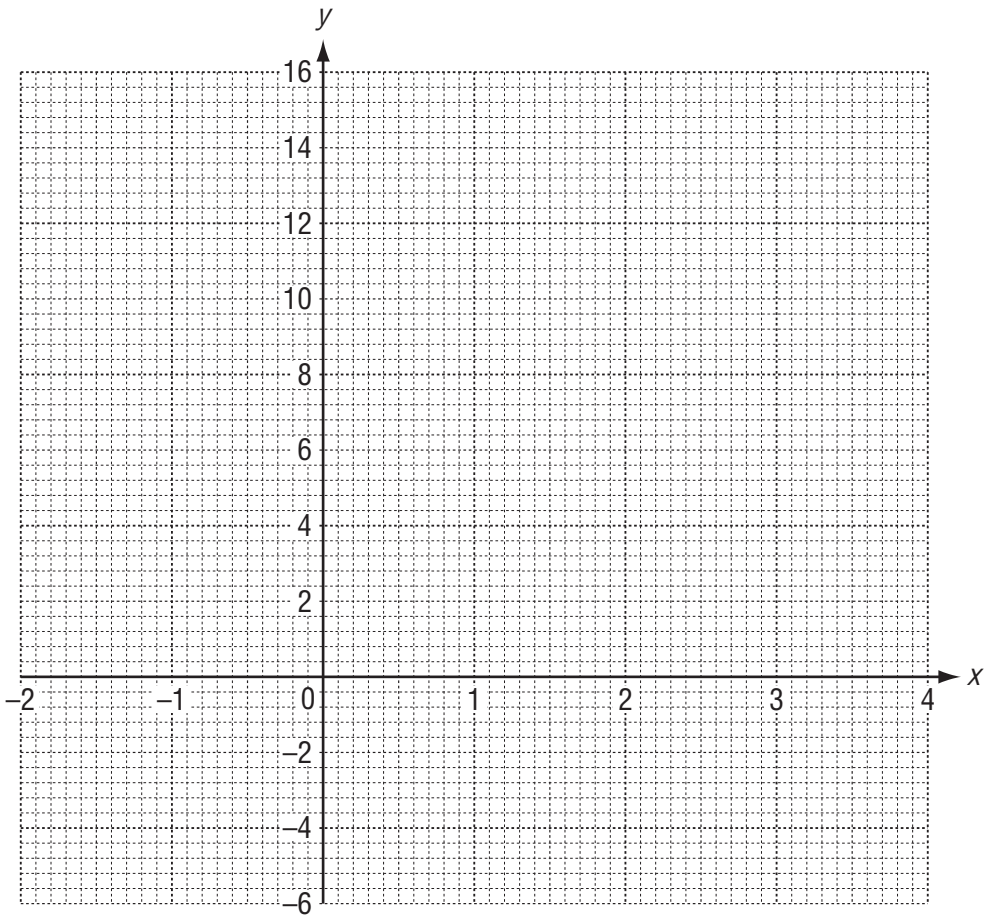
x	–1	0	1	2	3	4
$y = g(x)$		0	3		3	0

[2]

(c) On the grid, draw the graphs of

(i) $y = f(x)$ for $-2 \leqslant x \leqslant 4$, [3]

(ii) $y = g(x)$ for $-1 \leqslant x \leqslant 4$. [3]



(d) Use your graphs to solve the following equations.

(i) $f(x) = 10$

Answer(d)(i) $x =$ ..... [1]

(ii) $f(x) = g(x)$

Answer(d)(ii) $x =$ ..... or $x =$ ..... [2]

(iii) $f^{-1}(x) = 1.7$

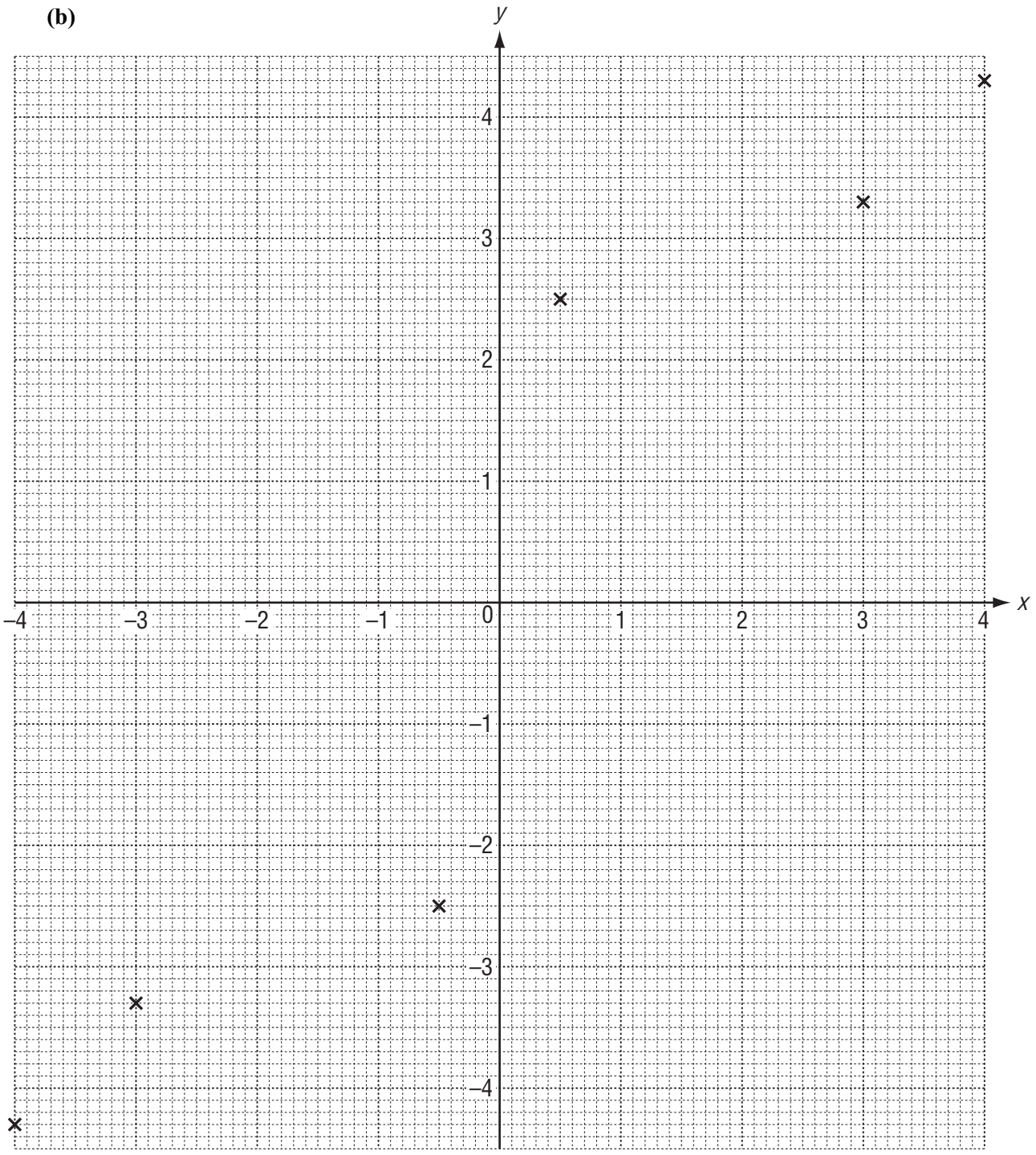
Answer(d)(iii) $x =$ ..... [1]

6 **(a)** Complete the table of values for $y = x + \frac{1}{x}$.

x	–4	–3	–2	–1	–0.5	0.5	1	2	3	4
y	–4.3	–3.3			–2.5	2.5			3.3	4.3

[2]

(b)



On the grid, draw the graph of $y = x + \frac{1}{x}$ for $-4 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 4$.

Six of the ten points have been plotted for you. [3]

(c) There are three integer values of k for which the equation $x + \frac{1}{x} = k$ has **no** solutions.
Write down these three values of k.

Answer(c) $k =$ ..... or $k =$ ..... or $k =$ ..... [2]

(d) Write down the ranges of x for which the gradient of the graph of $y = x + \frac{1}{x}$ is positive.

Answer(d) ..... [2]

(e) To solve the equation $x + \frac{1}{x} = 2x + 1$, a straight line can be drawn on the grid.

(i) Draw this line on the grid for $-2.5 \leqslant x \leqslant 1.5$. [2]

(ii) On the grid, show how you would find the solutions. [1]

(iii) Show how the equation $x + \frac{1}{x} = 2x + 1$ can be rearranged into the form $x^2 + bx + c = 0$
and find the values of b and c.

Answer(e)(iii) $b =$ .....

$c =$ ..... [3]

20 $f(x) = x^3$ $g(x) = 2x - 3$

(a) Find

(i) g(6),

Answer(a)(i) ..... [1]

(ii) f(2x).

Answer(a)(ii) ..... [1]

(b) Solve fg(x) = 125.

Answer(b) x = ... [3]

(c) Find the inverse function $g^{-1}(x)$.

Answer(c) $g^{-1}(x)$ = ... [2]

19 $f(x) = x^2$ $g(x) = 2^x$ $h(x) = 2x - 3$

(a) Find g(3).

Answer(a) ..... [1]

(b) Find hh(x) in its simplest form.

Answer(b) ..... [2]

(c) Find fg($x + 1$) in its simplest form.

Answer(c) ..... [2]

(b) The table shows some values of the function $y = x^2 - 2$.

x	–3	–2	–1	0	1	2	3
y	7		–1		–1		7

(i) Complete the table. [2]

(ii) On the grid, draw the graph of $y = x^2 - 2$ for $-3 \leqslant x \leqslant 3$. [4]

(iii) Use your graph to solve the equation $x^2 - 2 = 0$.

Answer(b)(iii) $x =$ ..... or $x =$ ..... [2]

(c) Write down the co-ordinates of the points where your graph meets the line AB.

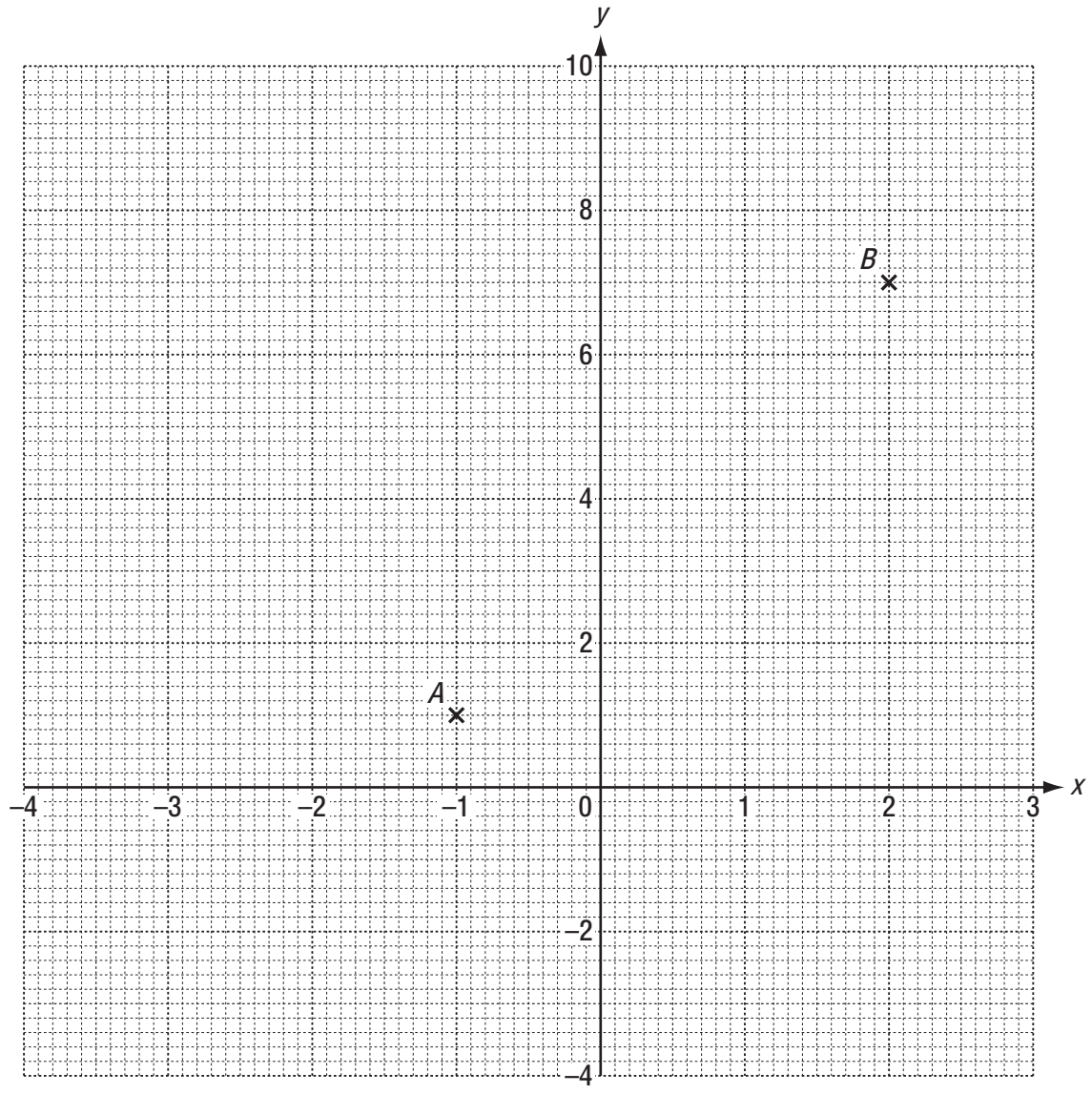
Answer(c)(..... , ...) and (..... , ...) [2]

7 **(a)** The table shows some values of the function $y = x^2 + x - 3$.

x	-4	-3	-2	-1	0	1	2	3
y	9	3		-3		-1		9

(i) Complete the table. [2]

(ii) On the grid, draw the graph of $y = x^2 + x - 3$ for $-4 \leqslant x \leqslant 3$.



[4]

(iii) Use your graph to solve the equation $x^2 + x - 3 = 0$.

Answer(a)(iii) $x =$ ..... or $x =$ ..... [2]

(b) **(i)** Draw the line of symmetry of the graph. [1]

(ii) Write down the equation of the line of symmetry.

Answer(b)(ii) ..... [1]

(c) Two points, A and B, are marked on the grid.

(i) Draw the straight line through the points A and B extending it to the edges of the grid. [1]

(ii) Write down the co-ordinates of the points of intersection of this line with $y = x^2 + x - 3$.

Answer(c)(ii) (..... , ...) and (..... , ...) [2]

(iii) Work out the gradient of the straight line through points A and B.

Answer(c)(iii) ..... [2]

(iv) Write down the equation of the straight line through points A and B, in the form $y = mx + c$.

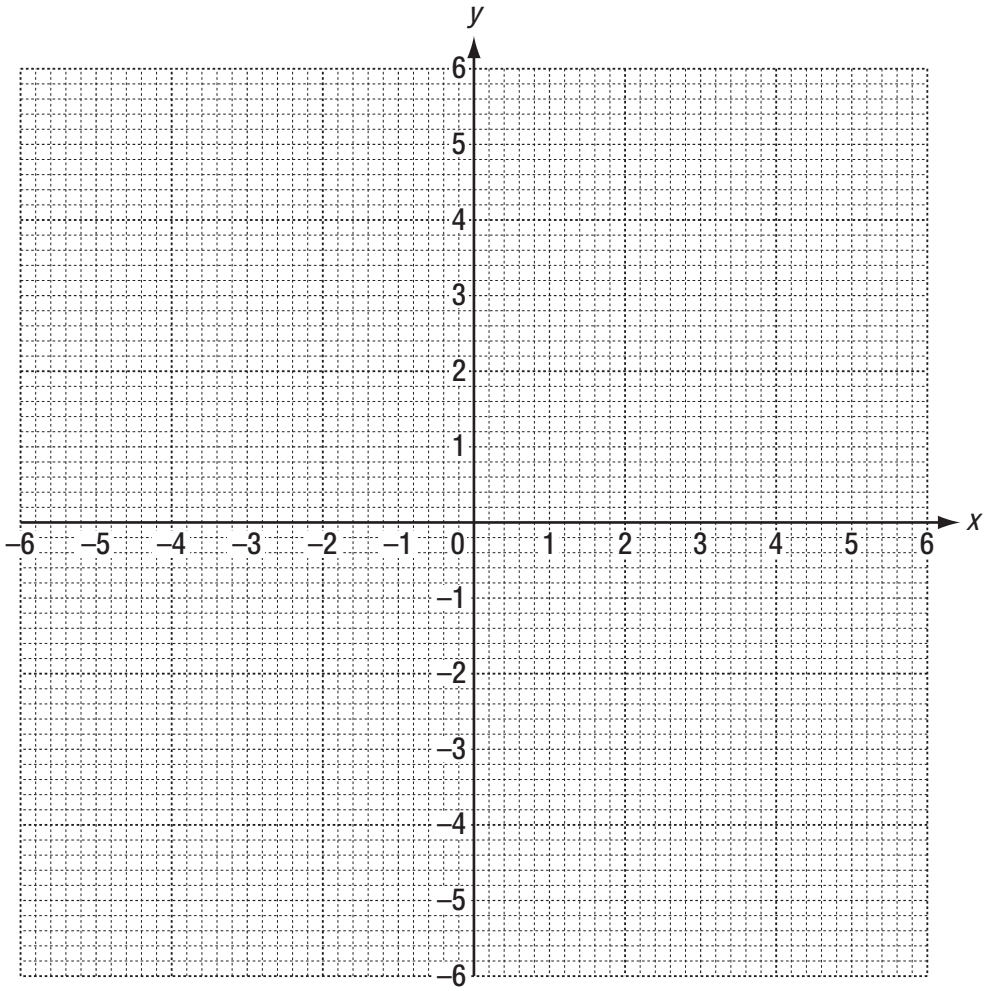
Answer(c)(iv) $y =$ ..... [2]

5 **(a)** **(i)** Complete the table for the function $y = \frac{6}{x}$, $x \neq 0$.

x	–6	–5	–4	–3	–2	–1	1	2	3	4	5	6
y	–1	–1.2		–2	–3	–6	6	3			1.2	1

[2]

(ii) On the grid, draw the graph of $y = \frac{6}{x}$ for $-6 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 6$.



[4]

(b) (i) Complete the table for the function $y = \frac{x^2}{2} - 2$.

x	–4	–3	–2	–1	0	1	2	3	4
y	6	2.5			–2			2.5	6

[2]

(ii) On the grid opposite, draw the graph of $y = \frac{x^2}{2} - 2$ for $-4 \leqslant x \leqslant 4$. [4]

(c) Write down the co-ordinates of the point of intersection of the two graphs.

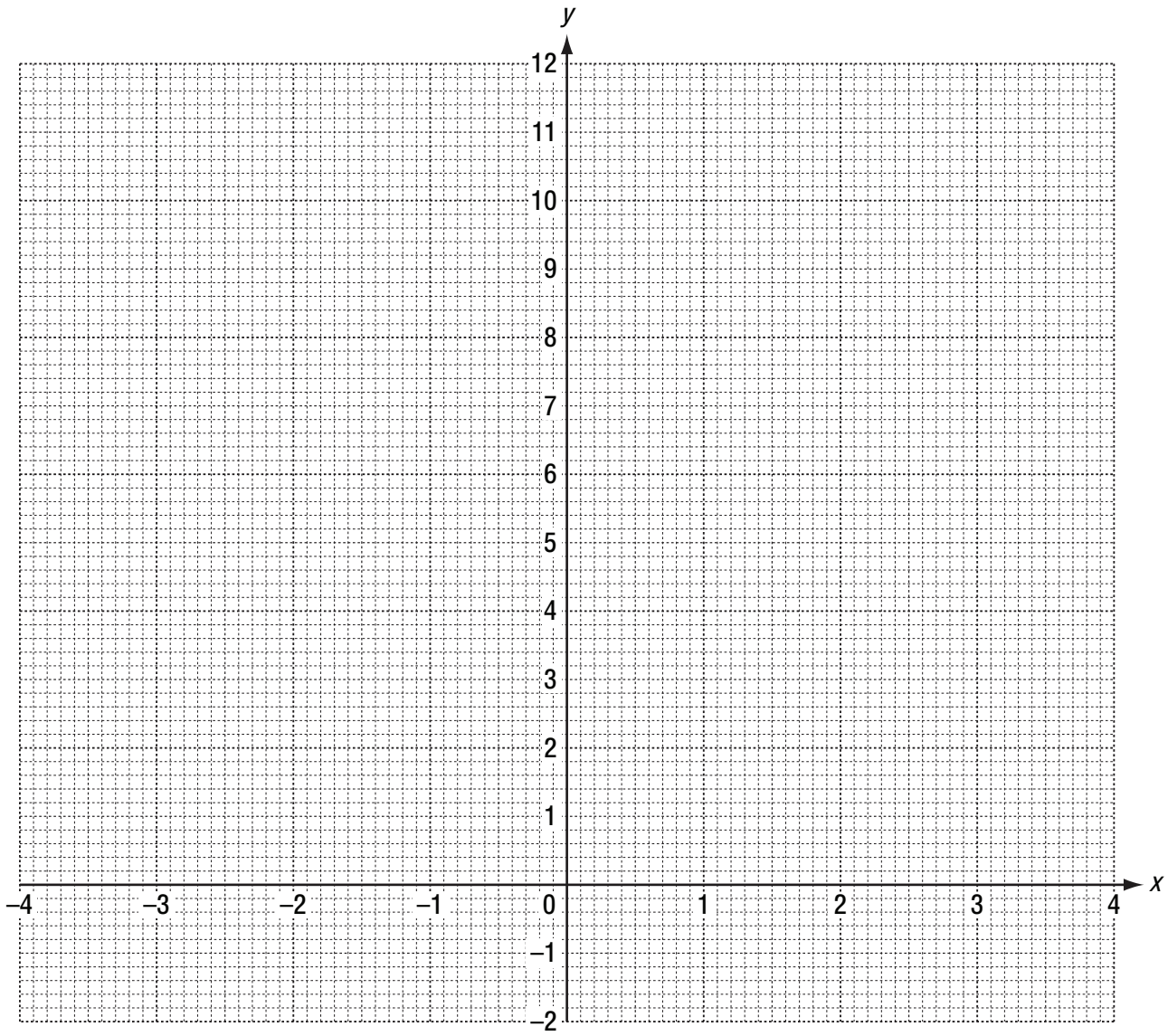
Answer(c)(..... , ...) [2]

7 **(a)** Complete the table of values for the equation $y = \frac{4}{x^2}$, $x \neq 0$.

x	-4	-3	-2	-1	-0.6		0.6	1	2	3	4
y	0.25	0.44			11.11			4.00		0.44	

[3]

(b) On the grid, draw the graph of $y = \frac{4}{x^2}$ for $-4 \leqslant x \leqslant -0.6$ and $0.6 \leqslant x \leqslant 4$.



[5]

(c) Use your graph to solve the equation $\frac{4}{x^2} = 6$.

Answer(c)x = ... or x = ... [2]

(d) By drawing a suitable tangent, estimate the gradient of the graph where $x = 1.5$.

Answer(d) ..... [3]

(e) **(i)** The equation $\frac{4}{x^2} - x + 2 = 0$ can be solved by finding the intersection of the graph of $y = \frac{4}{x^2}$ and a straight line.

Write down the equation of this straight line.

Answer(e)(i) ..... [1]

(ii) On the grid, draw the straight line from your answer to **part (e)(i)**. [2]

(iii) Use your graphs to solve the equation $\frac{4}{x^2} - x + 2 = 0$.

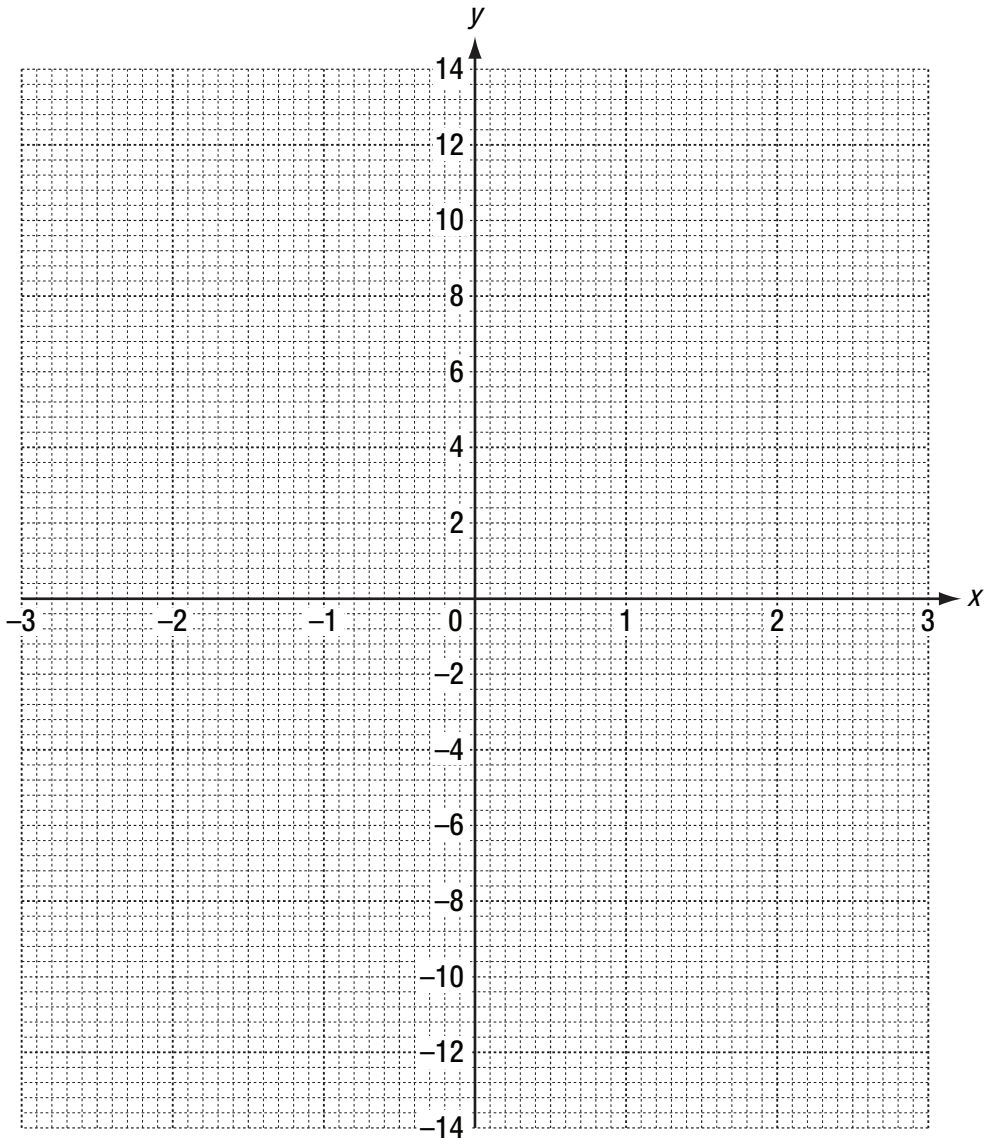
Answer(e)(iii) x = ... [1]

4 **(a)** Complete the table of values for the function $y = x^2 - \frac{3}{x}$, $x \neq 0$.

x	-3	-2	-1	-0.5	-0.25		0.25	0.5	1	2	3
y	10	5.5		6.3	12.1		-11.9			2.5	8

[3]

(b) Draw the graph of $y = x^2 - \frac{3}{x}$ for $-3 \leqslant x \leqslant -0.25$ and $0.25 \leqslant x \leqslant 3$.



[5]

(c) Use your graph to solve $x^2 - \frac{3}{x} = 7$.

Answer(c) $x =$ ..... or $x =$ ..... or $x =$ ..... [3]

(d) Draw the tangent to the curve where $x = -2$.
Use the tangent to calculate an estimate of the gradient of the curve where $x = -2$.

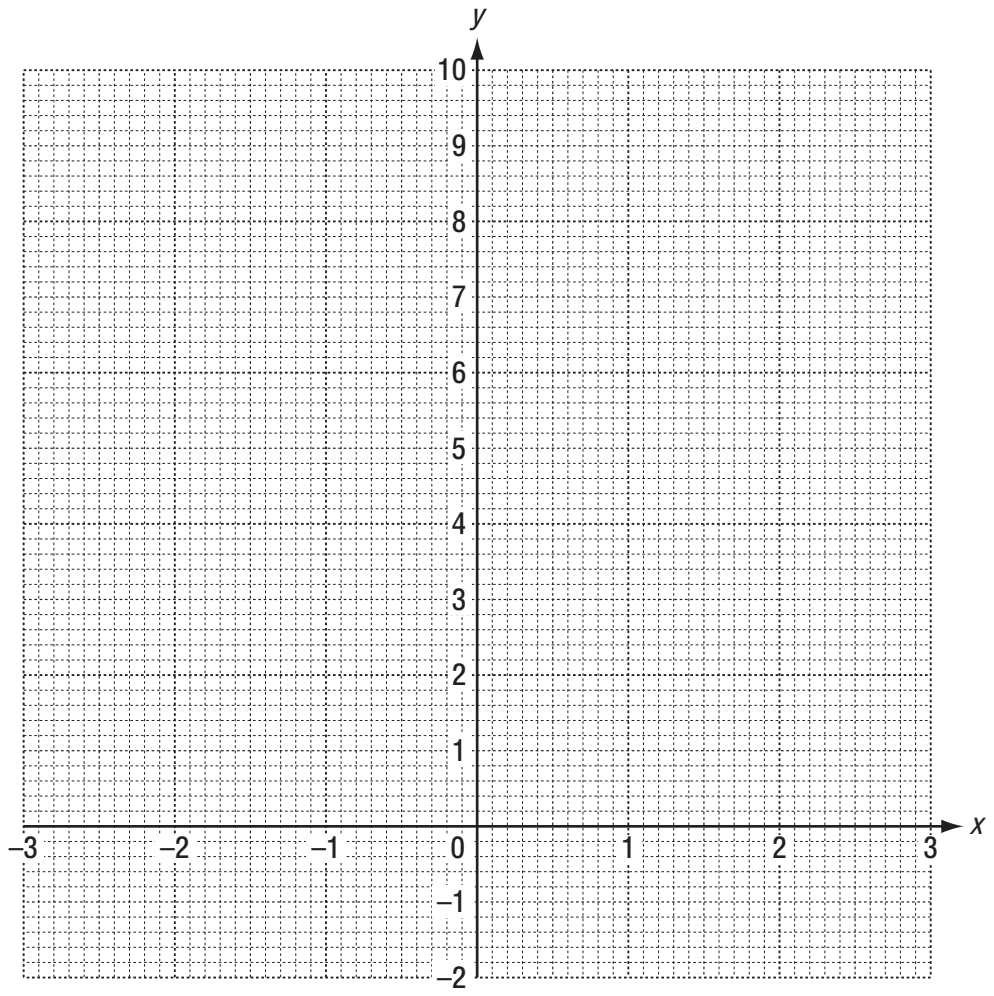
Answer(d) ..... [3]

5 **(a)** Complete the table of values for the function f(x), where $f(x) = x^2 + \frac{1}{x^2}$, $x \neq 0$.

x	-3	-2.5	-2	-1.5	-1	-0.5		0.5	1	1.5	2	2.5	3
f(x)		6.41		2.69		4.25		4.25		2.69		6.41	

[3]

(b) On the grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.5$ and $0.5 \leqslant x \leqslant 3$.



[5]

(c) (i) Write down the equation of the line of symmetry of the graph.

Answer(c)(i) ..... [1]

(ii) Draw the tangent to the graph of $y = f(x)$ where $x = -1.5$.
Use the tangent to estimate the gradient of the graph of $y = f(x)$ where $x = -1.5$.

Answer(c)(ii) ..... [3]

(iii) Use your graph to solve the equation $x^2 + \frac{1}{x^2} = 3$.

Answer(c)(iii) $x =$ ..... or $x =$ ..... or $x =$ ..... or $x =$ ..... [2]

(iv) Draw a suitable line on the grid and use your graphs to solve the equation $x^2 + \frac{1}{x^2} = 2x$.

Answer(c)(iv) $x =$ ..... or $x =$ ..... [3]

9 $f(x) = 3x + 1$ $g(x) = (x + 2)^2$

(a) Find the values of

(i) gf(2),

Answer(a)(i) ..... [2]

(ii) ff(0.5).

Answer(a)(ii) ..... [2]

(b) Find $f^{-1}(x)$, the inverse of $f(x)$.

Answer(b) ..... [2]

(c) Find $fg(x)$.

Give your answer in its simplest form.

Answer(c) ..... [2]

(d) Solve the equation $x^2 + f(x) = 0$.

Show all your working and give your answers correct to 2 decimal places.

Answer(d) $x =$ ..... or $x =$ ..... [4]

20 f: $x \to 2x - 1$ and g: $x \to x^2 - 1$.
Find, in their simplest forms,

(a) $f^{-1}(x)$,

Answer (a) $f^{-1}(x) =$ ..... [2]

(b) $gf(x)$.

Answer (b) $gf(x) =$ ..... [2]

5 Answer the whole of this question on a sheet of graph paper.

(a) The table gives values of $f(x) = \frac{24}{x^2} + x^2$ for $0.8 \leqslant x \leqslant 6$.

x	0.8	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6
$f(x)$	38.1	25	12.9	10	10.1	11.7	l	m	n	26	31	36.7

Calculate, correct to 1 decimal place, the values of l, m and n. [3]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 2 cm to represent 5 units on the y-axis, draw an x-axis for $0 \leqslant x \leqslant 6$ and a y-axis for $0 \leqslant y \leqslant 40$.

Draw the graph of $y = f(x)$ for $0.8 \leqslant x \leqslant 6$. [6]

(c) Draw the tangent to your graph at $x = 1.5$ and use it to calculate an estimate of the gradient of the curve at this point. [4]

(d) (i) Draw a straight line joining the points (0, 20) and (6, 32). [1]

(ii) Write down the equation of this line in the form $y = mx + c$. [2]

(iii) Use your graph to write down the x-values of the points of intersection of this line and the curve $y = f(x)$. [2]

(iv) Draw the tangent to the curve which has the same gradient as your line in **part d(i)**. [1]

(v) Write down the equation for the tangent in **part d(iv)**. [2]

---

6 (a) On 1st January 2000, Ashraf was x years old.
Bukki was 5 years older than Ashraf and Claude was twice as old as Ashraf.

(i) Write down in terms of x, the ages of Bukki and Claude on 1st January 2000. [2]

(ii) Write down in terms of x, the ages of Ashraf, Bukki and Claude on 1st January 2002. [1]

(iii) The product of Claude's age and Ashraf's age on 1st January **2002** is the same as the square of Bukki's age on 1st January **2000**.
Write down an equation in x and show that it simplifies to $x^2 - 4x - 21 = 0$. [4]

(iv) Solve the equation $x^2 - 4x - 21 = 0$. [2]

(v) How old was Claude on 1st January 2002? [1]

(b) Claude's height, h metres, is one of the solutions of $h^2 + 8h - 17 = 0$.

(i) Solve the equation $h^2 + 8h - 17 = 0$.

Show all your working and give your answers correct to 2 decimal places. [4]

(ii) Write down Claude's height, to the nearest centimetre. [1]

---

4 **Answer the whole of this question on a sheet of graph paper.**

t	0	1	2	3	4	5	6	7
f(t)	0	25	37.5	43.8	46.9	48.4	49.2	49.6

(a) Using a scale of 2 cm to represent 1 unit on the horizontal t-axis and 2 cm to represent 10 units on the y-axis, draw axes for $0 \leqslant t \leqslant 7$ and $0 \leqslant y \leqslant 60$.
Draw the graph of the curve $y = \text{f}(t)$ using the table of values above. [5]

(b) $\text{f}(t) = 50(1 - 2^{-t})$.

(i) Calculate the value of f(8) and the value of f(9). [2]

(ii) Estimate the value of f(t) when t is large. [1]

(c) **(i)** Draw the tangent to $y = \text{f}(t)$ at $t = 2$ and use it to calculate an estimate of the gradient of the curve at this point. [3]

(ii) The function f(t) represents the speed of a particle at time t.
Write down what quantity the gradient gives. [1]

(d) **(i)** On the same grid, draw $y = \text{g}(t)$ where $\text{g}(t) = 6t + 10$, for $0 \leqslant t \leqslant 7$. [2]

(ii) Write down the range of values for t where $\text{f}(t) > \text{g}(t)$. [2]

(iii) The function g(t) represents the speed of a second particle at time t.
State whether the first or second particle travels the greater distance for $0 \leqslant t \leqslant 7$.
You **must** give a reason for your answer. [2]

5

Adam writes his name on four red cards and Daniel writes his name on six white cards.

(a) One of the ten cards is chosen at random. Find the probability that

(i) the letter on the card is **D**, [1]

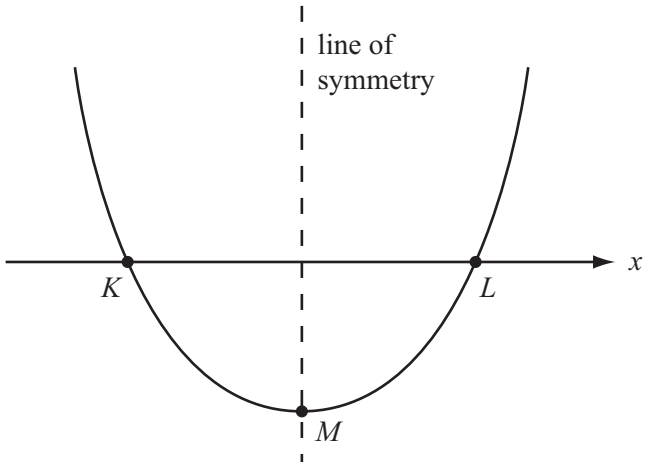
(ii) the card is red, [1]

(iii) the card is red **or** the letter on the card is **D**, [1]

(iv) the card is red **and** the letter on the card is **D**, [1]

(v) the card is red **and** the letter on the card is **N**. [1]

7 A sketch of the graph of the quadratic function $y = px^2 + qx + r$ is shown in the diagram.



The graph cuts the x-axis at K and L.
The point M lies on the graph and on the line of symmetry.

(a) When $p = 1, \quad q = -2, \quad r = -3$, find

(i) the y-coordinate of the point where $x = 4$, [1]

(ii) the coordinates of K and L, [3]

(iii) the coordinates of M. [2]

(b) Describe how the above sketch of the graph would change in each of the following cases.

(i) p is negative. [1]

(ii) $p = 1, \quad q = r = 0$. [1]

(c) Another quadratic function is $y = ax^2 + bx + c$.

(i) Its graph passes through the origin.
Write down the value of c. [1]

(ii) The graph also passes through the points (3, 0) and (4, 8).
Find the values of a and b. [4]

8 **(a)** The technical data of a car includes the following information.

Type of road	Petrol used per 100 km
Main roads	9.2 litres
Other roads	8.0 litres

(i) How much petrol is used on a journey of 350 km on a main road? [1]

(ii) On other roads, how far can the car travel on 44 litres of petrol? [1]

(iii) A journey consists of 200 km on a main road and 160 km on other roads.

(a) How much petrol is used? [2]

(b) Work out the amount of petrol used per 100 km of this journey. [1]

(b) A model of a car has a scale of 1 : 25.

(i) The length of the car is 3.95 m.
Calculate the length of the model.
Give your answer in centimetres. [3]

(ii) The painted surface area of the model is 128 cm^2.
Calculate the painted surface area of the car, giving your answer in square centimetres. [2]

(iii) The size of the luggage space of the car is 250 litres.
Calculate the size of the luggage space of the model, giving your answer in millilitres. [3]

---

9 **(a)** $f(x) = 2 - 3x$ and $g(x) = x^2$.

(i) Solve the equation $f(x) = 7 - x$. [2]

(ii) Find $f^{-1}(x)$. [2]

(iii) Find the value of $gf(2) - fg(2)$. [3]

(iv) Find $fg(x)$. [1]

(b) $h(x) = x^x$.

(i) Find the value of $h(2)$. [1]

(ii) Find the value of $h(-3)$, giving your answer as a fraction. [1]

(iii) Find the value of $h(7.5)$, giving your answer in standard form. [2]

(iv) $h(-0.5)$ is not a real number. Explain why. [1]

(v) Find the integer value for which $h(x) = 3125$. [1]

(d) **On the same grid**, draw the graph of $y = 2x - 5$ for $-3 \leqslant x \leqslant 3$. [2]

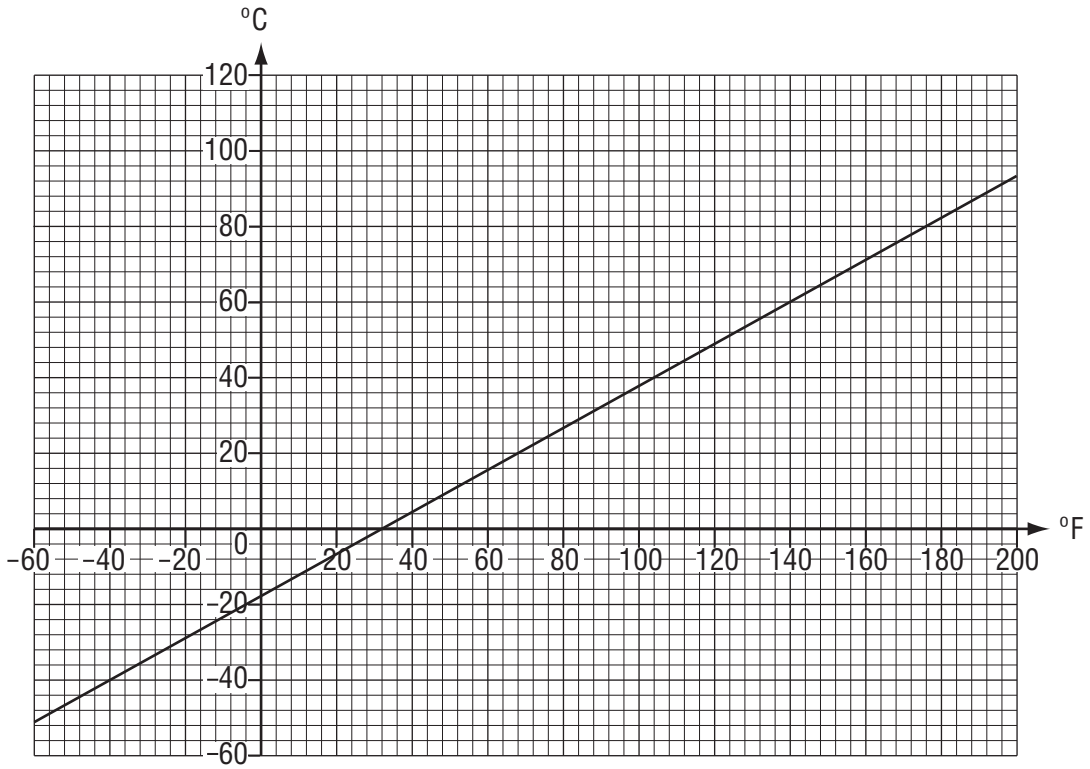
(e) **(i)** Use your graphs to find solutions of the equation $1 - \frac{1}{x^2} = 2x - 5$. [3]

(ii) Rearrange $1 - \frac{1}{x^2} = 2x - 5$ into the form $ax^3 + bx^2 + c = 0$, where a, b and c are integers. [2]

(f) **(i)** Draw a tangent to the graph of $y = f(x)$ which is parallel to the line $y = 2x - 5$. [1]

(ii) Write down the equation of this tangent. [2]

14 The graph drawn below shows the conversion of temperatures in degrees Fahrenheit (°F) to temperatures in degrees Celsius (°C).



(a) The temperature of a room is 20 °C. What is the temperature in Fahrenheit?

Answer(a) ..... [1]

(b) A liquid has a boiling point of 176 °F. What is the temperature in Celsius?

Answer(b) ..... [1]

(c) Find T when T °C = T °F.

Answer(c) T = ... [1]

---

15 f: $x \mapsto 5 - 3x$.

(a) Find f(−1).

Answer(a) ..... [1]

(b) Find $\mathrm{f}^{-1}(x)$.

Answer(b) ..... [2]

(c) Find $\mathrm{f}\mathrm{f}^{-1}(8)$.

Answer(c) ..... [1]

---

4 Answer the whole of this question on a sheet of graph paper.

$$f(x) = 3x - \frac{1}{x^2} + 3, \quad x \neq 0.$$

(a) The table shows some values of $f(x)$.

x	−3	−2.5	−2	−1.5	−1	−0.5	−0.4	−0.3	0.3	0.4	0.5	1	1.5	2	2.5	3
$f(x)$	p	−4.7	−3.3	−1.9	−1	−2.5	−4.5	−9.0	−7.2	−2.1	0.5	q	7.1	8.8	10.3	r

Find the values of p, q and r. [3]

(b) Draw axes using a scale of 1 cm to represent 0.5 units for $-3 \leqslant x \leqslant 3$ and 1 cm to represent 2 units for $-10 \leqslant y \leqslant 12$. [1]

(c) On your grid, draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.3$ and $0.3 \leqslant x \leqslant 3$. [5]

(d) Use your graph to solve the equations

(i) $3x - \frac{1}{x^2} + 3 = 0$, [1]

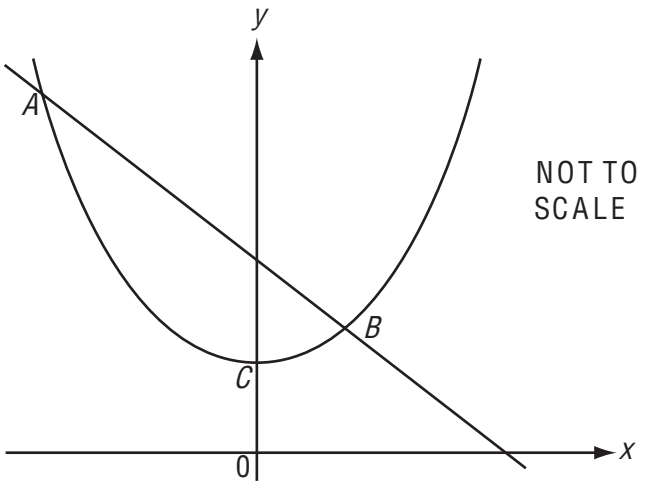
(ii) $3x - \frac{1}{x^2} + 7 = 0$. [3]

(e) $g(x) = 3x + 3$.
On the same grid, draw the graph of $y = g(x)$ for $-3 \leqslant x \leqslant 3$. [2]

(f) **(i)** Describe briefly what happens to the graphs of $y = f(x)$ and $y = g(x)$ for large positive or negative values of x. [1]

(ii) Estimate the gradient of $y = f(x)$ when $x = 100$. [1]

3



The diagram shows a sketch of $y = x^2 + 1$ and $y = 4 - x$.

(a) Write down the co-ordinates of

(i) the point C, [1]

(ii) the points of intersection of $y = 4 - x$ with each axis. [2]

(b) Write down the gradient of the line $y = 4 - x$. [1]

(c) Write down the range of values of x for which the gradient of the graph of $y = x^2 + 1$ is negative. [1]

(d) The two graphs intersect at A and B.

Show that the x co-ordinates of A and B satisfy the equation $x^2 + x - 3 = 0$. [1]

(e) Solve the equation $x^2 + x - 3 = 0$, giving your answers correct to 2 decimal places. [4]

(f) Find the co-ordinates of the mid-point of the straight line AB. [2]

3 Answer the whole of this question on a sheet of graph paper.

The table shows some of the values of the function $f(x) = x^2 - \frac{1}{x}$, $x \neq 0$.

x	−3	−2	−1	−0.5	−0.2	0.2	0.5	1	2	3
y	9.3	4.5	2.0	2.3	p	−5.0	−1.8	q	3.5	r

(a) Find the values of p, q and r, correct to 1 decimal place. [3]

(b) Using a scale of 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw an x-axis for $-3 \leqslant x \leqslant 3$ and a y-axis for $-6 \leqslant y \leqslant 10$.

Draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$. [6]

(c) (i) By drawing a suitable straight line, find the three values of x where $f(x) = -3x$. [3]

(ii) $x^2 - \frac{1}{x} = -3x$ can be written as $x^3 + ax^2 + b = 0$.

Find the values of a and b. [2]

(d) Draw a tangent to the graph of $y = f(x)$ at the point where $x = -2$.

Use it to estimate the gradient of $y = f(x)$ when $x = -2$. [3]

22 $f(x) = 4x + 1$ $g(x) = x^3 + 1$ $h(x) = \frac{2x+1}{3}$

(a) Find the value of gf(0).

Answer(a) ..... [2]

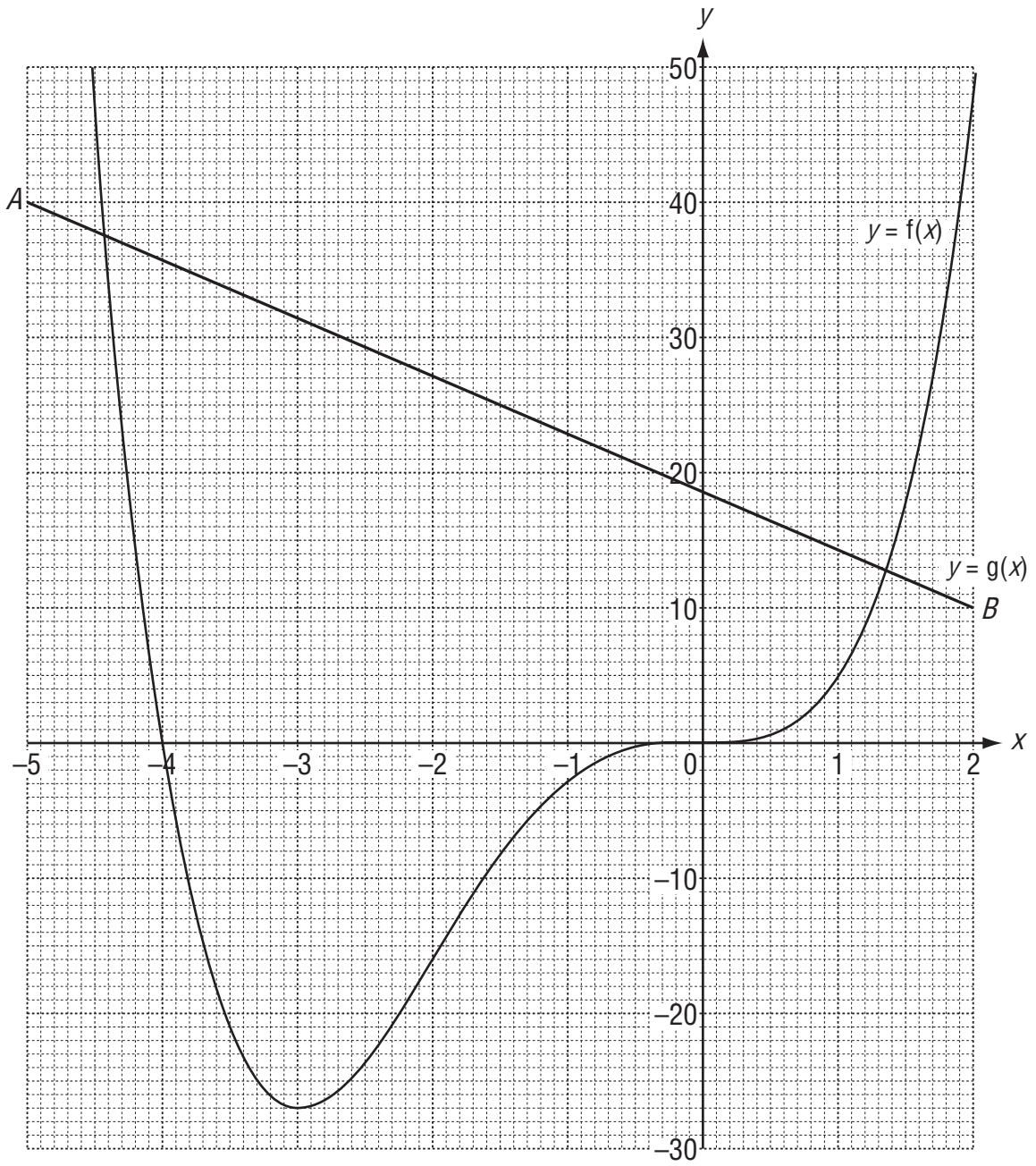
(b) Find fg(x). Simplify your answer.

Answer(b) ..... [2]

(c) Find $h^{-1}(x)$.

Answer(c) ..... [2]

6



The graphs of $y = f(x)$ and $y = g(x)$ are shown above.

(a) Find the value of

(i) f(–2),

Answer(a)(i) ..... [1]

(ii) g(0).

Answer(a)(ii) ..... [1]

(b) Use the graphs to solve

(i) the equation f(x) = 20,

Answer(b)(i) x = ... or x = ... [2]

(ii) the equation f(x) = g(x),

Answer(b)(ii) x = ... or x = ... [2]

(iii) the inequality f(x) < g(x).

Answer(b)(iii) ..... [1]

(c) Use the points A and B to find the gradient of $y = g(x)$ as an exact fraction.

Answer(c) ..... [2]

(d) On the grid, draw the graph of $y = g(x) - 10$. [2]

(e) **(i)** Draw the tangent to the graph of $y = f(x)$ at $(-3, -27)$. [1]

(ii) Write down the equation of this tangent.

Answer(e)(ii) ..... [1]

(f) A region, R, contains points whose co-ordinates satisfy the inequalities

$$-3 \leqslant x \leqslant -2, \qquad y \leqslant 40 \qquad \text{and} \qquad y \geqslant g(x).$$

On the grid, draw suitable lines and label this region R. [2]

8 **(a)** $f(x) = 2x - 1$ $g(x) = x^2$

Work out

(i) f(2),

Answer(a)(i) ..... [1]

(ii) g(− 2),

Answer(a)(ii) ..... [1]

(iii) ff(x) in its simplest form,

Answer(a)(iii) ff(x) = ... [2]

(iv) $f^{-1}(x)$, the inverse of f(x),

Answer(a)(iv) $f^{-1}(x)$ = ... [2]

(v) x when gf(x) = 4.

Answer(a)(v) x = ... or x = ... [4]

(b) y is **inversely** proportional to x and $y = 8$ when $x = 2$.

Find,

(i) an equation connecting y and x,

Answer(b)(i) ..... [2]

(ii) y when $x = \frac{1}{2}$.

Answer(b)(ii) y = ... [1]

2 $f(x) = 6 + x^2$ $g(x) = 4x - 1$

(a) Find

(i) g(3),

Answer(a)(i) ..... [1]

(ii) f (–4).

Answer(a)(ii) ..... [1]

(b) Find the inverse function $g^{-1}(x)$.

Answer(b) $g^{-1}(x) =$ ..... [2]

(c) Find $fg(x)$ in its simplest form.

Answer(c) $fg(x) =$ ..... [3]

(d) Solve the equation $gg(x) = 3$.

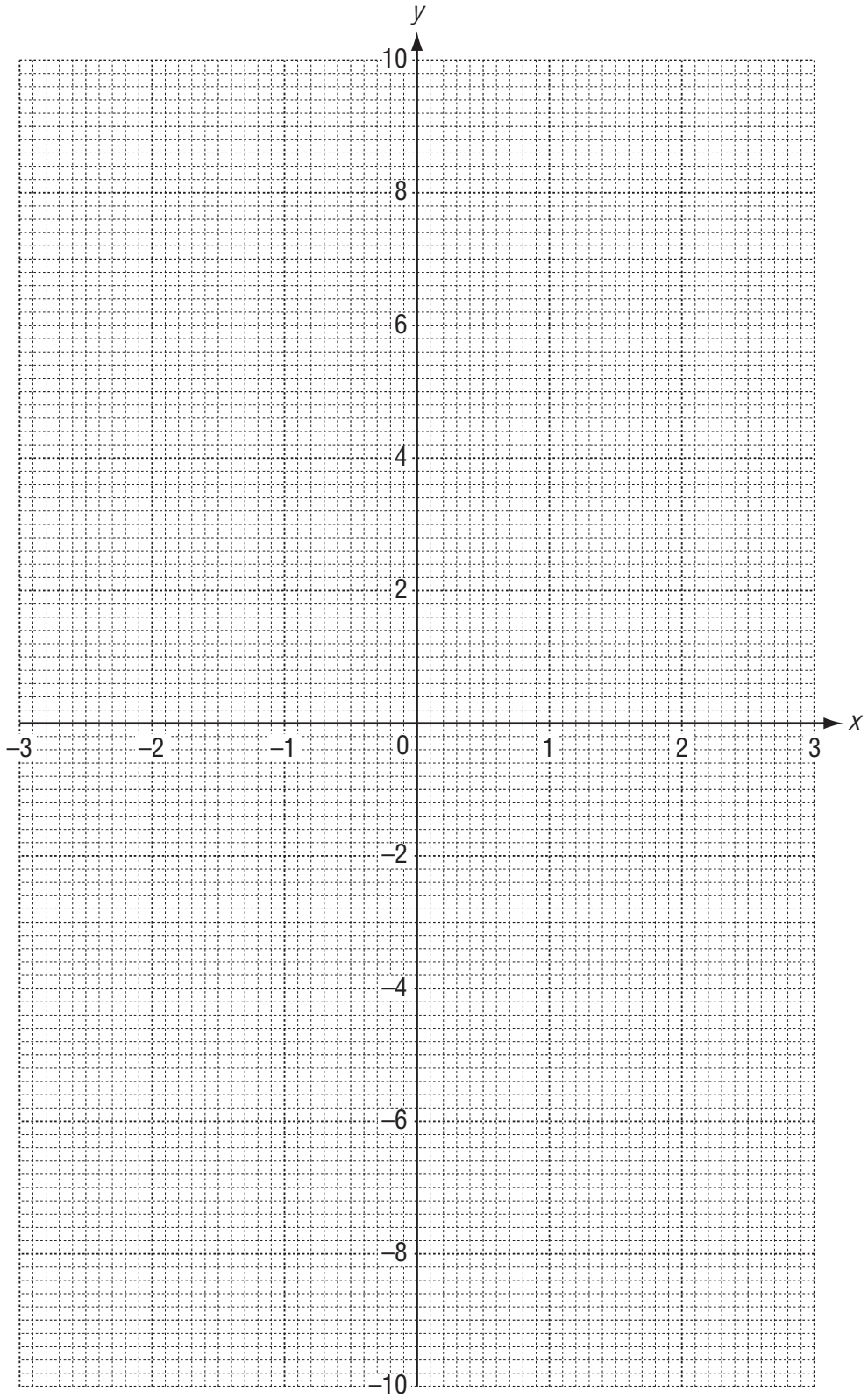
Answer(d) $x =$ ..... [3]

7 **(a)** Complete the table for the function $f(x) = \frac{2}{x} - x^2$.

x	–3	–2	–1	–0.5	–0.2		0.2	0.5	1	2	3
f(x)	–9.7	–5			–10.0		10.0	3.75	1		–8.3

[3]

(b) On the grid draw the graph of $y = f(x)$ for $-3 \leqslant x \leqslant -0.2$ and $0.2 \leqslant x \leqslant 3$.



[5]

(c) Use your graph to

(i) solve f(x) = 2,

Answer(c)(i) x = ... [1]

(ii) find a value for k so that f(x) = k has 3 solutions.

Answer(c)(ii) k = ... [1]

(d) Draw a suitable line on the grid and use your graphs to solve the equation $\frac{2}{x} - x^2 = 5x$.

Answer(d) x = ... or x = ... [3]

(e) Draw the tangent to the graph of y = f(x) at the point where $x = -2$.

Use it to calculate an estimate of the gradient of y = f(x) when $x = -2$.

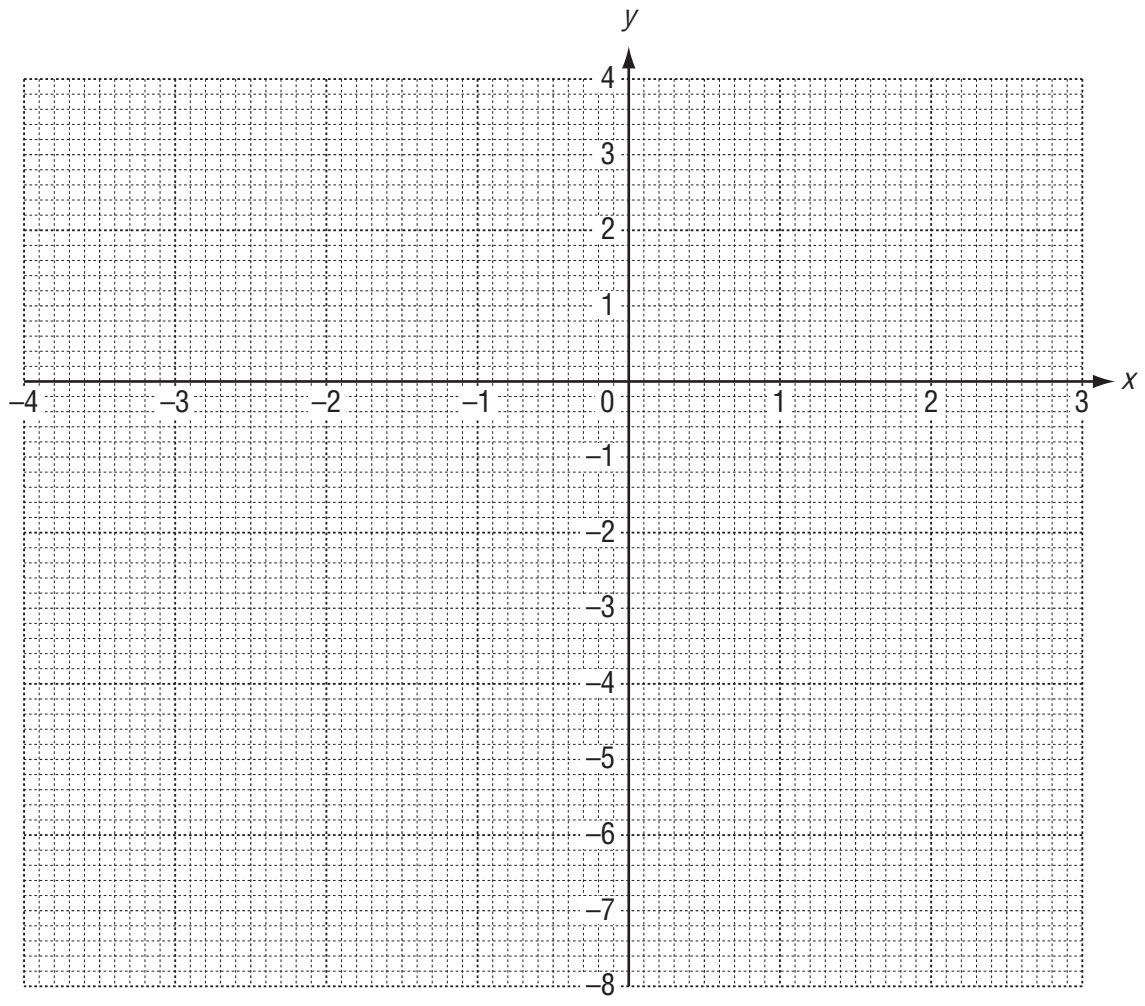
Answer(e) ..... [3]

7 **(a)** Complete the table for the function $f(x) = \frac{x^3}{10} + 1$.

x	–4	–3	–2	–1	0	1	2	3
f(x)		–1.7	0.2	0.9	1	1.1	1.8	

[2]

(b) On the grid, draw the graph of $y = f(x)$ for $-4 \leqslant x \leqslant 3$.



[4]

(c) Complete the table for the function $g(x) = \frac{4}{x}$, $x \neq 0$.

x	–4	–3	–2	–1	1	2	3
g(x)	–1	–1.3				2	1.3

[2]

(d) On the grid, draw the graph of $y = g(x)$ for $-4 \leqslant x \leqslant -1$ and $1 \leqslant x \leqslant 3$. [3]

(e) (i) Use your graphs to solve the equation $\frac{x^3}{10} + 1 = \frac{4}{x}$.

Answer(e)(i) $x =$ ..... or $x =$ ..... [2]

(ii) The equation $\frac{x^3}{10} + 1 = \frac{4}{x}$ can be written as $x^4 + ax + b = 0$.

Find the values of a and b.

Answer(e)(ii) $a =$ .....

$b =$ ..... [2]