Mensuration
2002 - 2011

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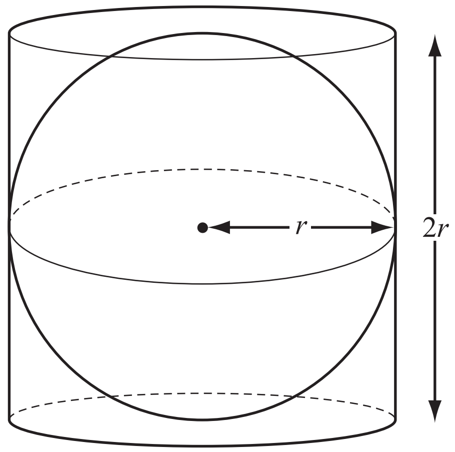
14



The sphere of radius r fits exactly inside the cylinder of radius r and height $2r$.
Calculate the percentage of the cylinder occupied by the sphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

Answer .... % [3]

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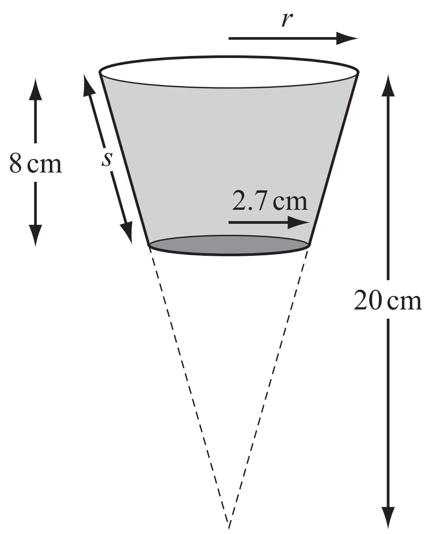
15 $ap = px + c$

Write p in terms of a, c and x.

Answer $p =$ .... [3]

4



The diagram shows a plastic cup in the shape of a cone with the end removed.
The vertical height of the cone in the diagram is 20 cm.
The height of the cup is 8 cm.
The base of the cup has radius 2.7 cm.

(a) **(i)** Show that the radius, r, of the circular top of the cup is 4.5 cm.

Answer(a)(i)

[2]

(ii) Calculate the volume of water in the cup when it is full.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(a)(ii) .... cm^3 [4]

(b) **(i)** Show that the slant height, s, of the cup is 8.2 cm.

Answer(b)(i)

[3]

(ii) Calculate the curved surface area of the outside of the cup.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

Answer(b)(ii) .... cm^2 [5]

(b) John wants to estimate the value of π.
He measures the circumference of a circular pizza as 105 cm and its diameter as 34 cm, both correct to the nearest centimetre.

Calculate the lower bound of his estimate of the value of π.
Give your answer correct to 3 decimal places.

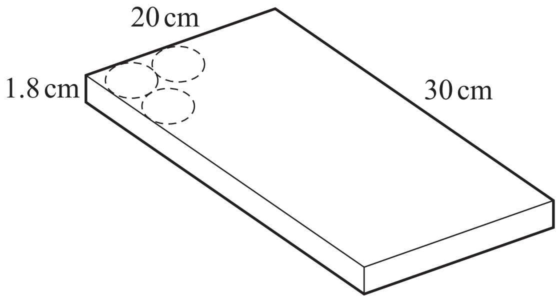
Answer(b) .... [4]

(c) The volume of a cylindrical can is $550\,\text{cm}^3$, correct to the nearest $10\,\text{cm}^3$.
The height of the can is 12 cm correct to the nearest centimetre.

Calculate the upper bound of the radius of the can.
Give your answer correct to 3 decimal places.

Answer(c) .... cm [5]

(c)



The $1080\,\text{cm}^3$ of dough is then rolled out to form a cuboid $20\,\text{cm} \times 30\,\text{cm} \times 1.8\,\text{cm}$.

Boris cuts out circular biscuits of diameter 5 cm.

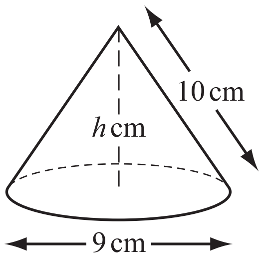
(i) How many whole biscuits can he cut from this cuboid?

Answer(c)(i) .... [1]

(ii) Calculate the volume of dough left over.

Answer(c)(ii) .... cm^3 [3]

6



NOT TO SCALE

A solid cone has diameter 9 cm, slant height 10 cm and vertical height h cm.

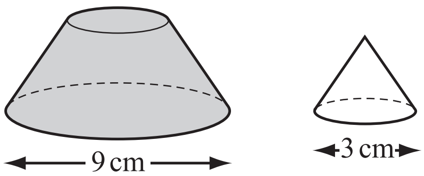
(a) **(i)** Calculate the curved surface area of the cone.
[The curved surface area, A, of a cone, radius r and slant height l is $A = \pi r l$.]

Answer(a)(i) .... cm^2 [2]

(ii) Calculate the value of h, the vertical height of the cone.

Answer(a)(ii) h = ... [3]

(b)



NOT TO SCALE

Sasha cuts off the top of the cone, making a smaller cone with diameter 3 cm.
This cone is **similar** to the original cone.

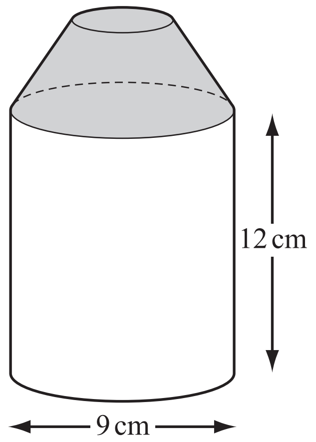
(i) Calculate the **vertical** height of this small cone.

Answer(b)(i) .... cm [2]

(ii) Calculate the curved surface area of this small cone.

Answer(b)(ii) .... cm^2 [2]

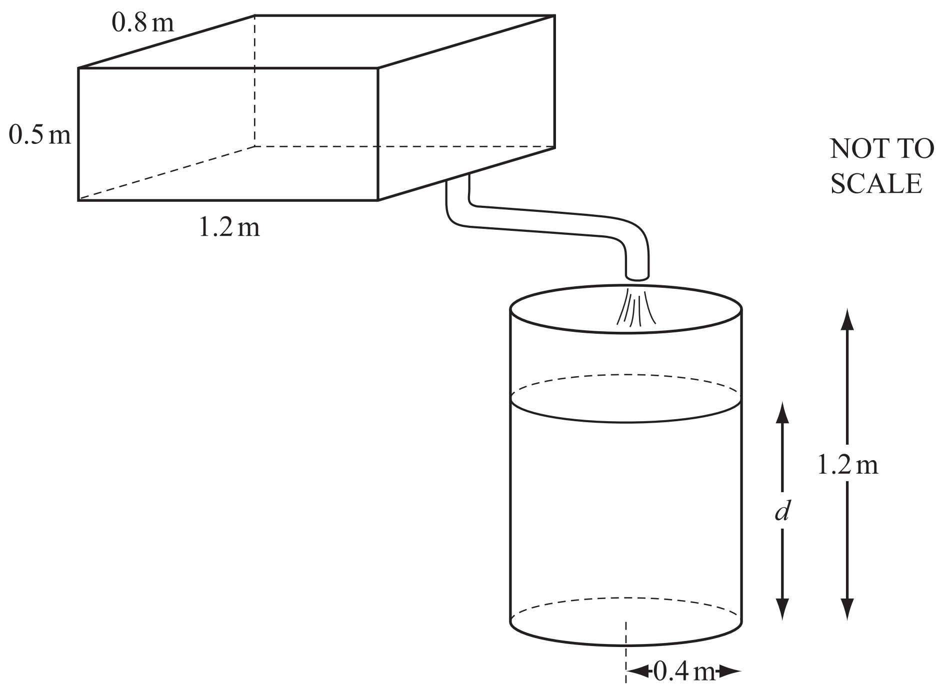
(c)



The shaded solid from **part (b)** is joined to a solid cylinder with diameter 9 cm and height 12 cm.
Calculate the **total** surface area of the whole solid.

Answer(c) .... cm^2 [5]

1



A rectangular tank measures 1.2 m by 0.8 m by 0.5 m.

(a) Water flows from the full tank into a cylinder at a rate of 0.3 m^3/min.

Calculate the time it takes for the full tank to empty.
Give your answer in minutes and seconds.

Answer(a) .... min .... s [3]

(b) The radius of the cylinder is 0.4 m.

Calculate the depth of water, *d*, when all the water from the rectangular tank is in the cylinder.

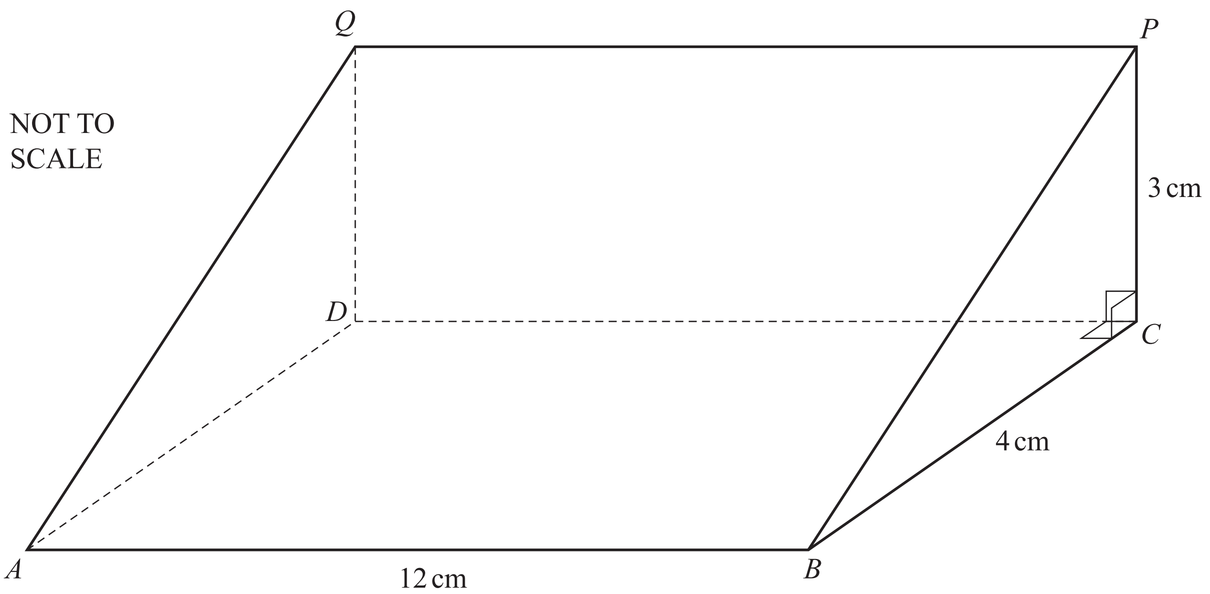
Answer(b) d = ... m [3]

(c) The cylinder has a height of 1.2 m and is open at the top.
The inside surface is painted at a cost of \$2.30 per m^2.

Calculate the cost of painting the inside surface.

Answer(c) \$ .... [4]

6



The diagram shows a triangular prism of length 12 cm.

The rectangle *ABCD* is horizontal and the rectangle *DCPQ* is vertical.

The cross-section is triangle *PBC* in which angle *BCP* = 90°, *BC* = 4 cm and *CP* = 3 cm.

(a) **(i)** Calculate the length of *AP*.

Answer(a)(i) *AP* = ... cm [3]

(ii) Calculate the angle of elevation of *P* from *A*.

Answer(a)(ii) .... [2]

(b) **(i)** Calculate angle PBC.

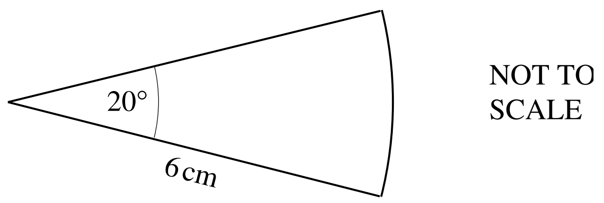
Answer(b)(i) Angle PBC = ... [2]

(ii) X is on BP so that angle $BXC = 120°$.

Calculate the length of XC.

Answer(b)(ii) XC = ... cm [3]

8 **(a)** A sector of a circle, radius 6 cm, has an angle of 20°.

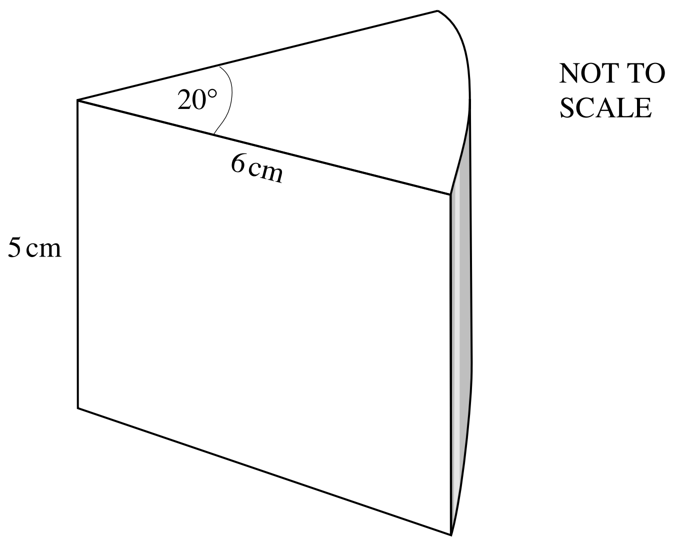


Calculate

(i) the area of the sector, [2]

(ii) the arc length of the sector. [2]

(b)



A whole cheese is a cylinder, radius 6 cm and height 5 cm.
The diagram shows a slice of this cheese with sector angle 20°.

Calculate

(i) the volume of the slice of cheese, [2]

(ii) the **total** surface area of the slice of cheese. [4]

(c) The radius, r, and height, h, of cylindrical cheeses vary but the volume remains constant.

(i) Which one of the following statements A, B, C or D is true?

A: h is proportional to r.

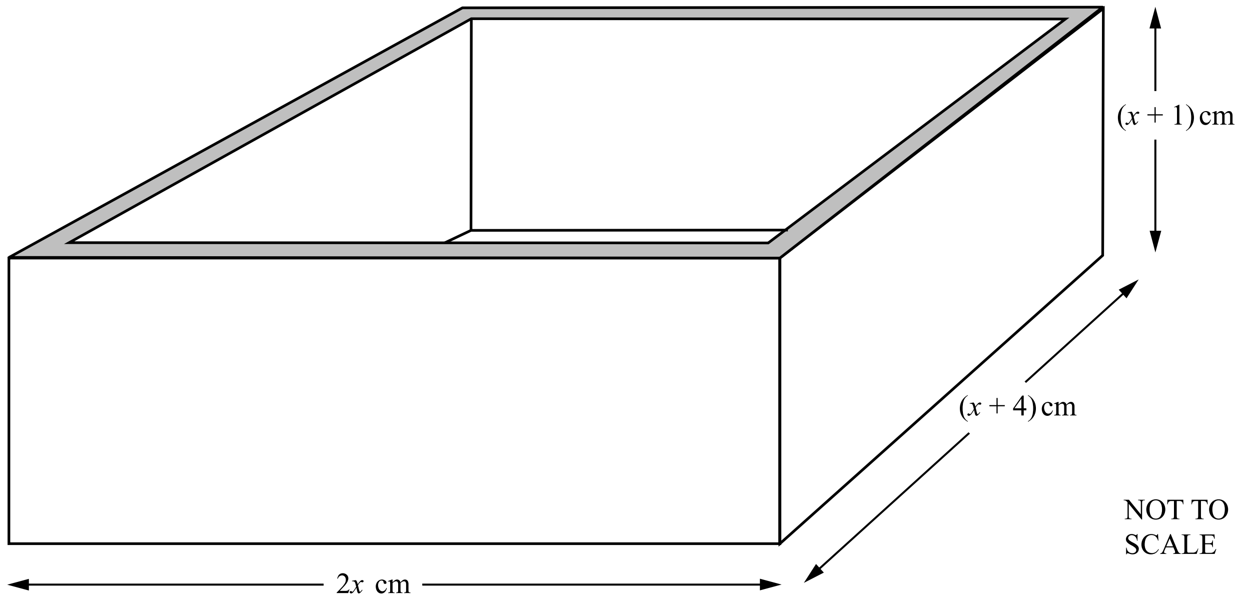
B: h is proportional to r^2.

C: h is inversely proportional to r.

D: h is inversely proportional to r^2. [2]

(ii) What happens to the height h of the cylindrical cheese when the volume remains constant but the radius is doubled? [2]

6



A rectangular-based **open** box has **external** dimensions of $2x$ cm, $(x + 4)$ cm and $(x + 1)$ cm.

(a) **(i)** Write down the volume of a cuboid with these dimensions. [1]

(ii) Expand and simplify your answer. [1]

(b) The box is made from wood 1 cm thick.

(i) Write down the **internal** dimensions of the box in terms of x. [3]

(ii) Find the volume of the **inside** of the box and show that the volume of the wood is $8x^2 + 12x$ cubic centimetres. [3]

(c) The volume of the wood is 1980 cm^3.

(i) Show that $2x^2 + 3x - 495 = 0$ and solve this equation. [5]

(ii) Write down the **external** dimensions of the box. [2]

9 **(a)**

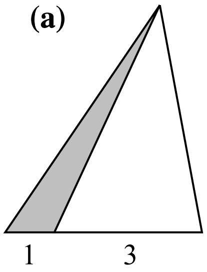


Diagram 1

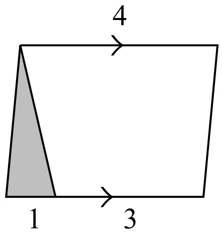


Diagram 2

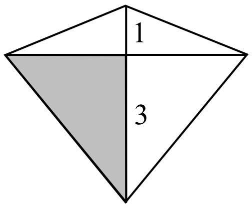


Diagram 3

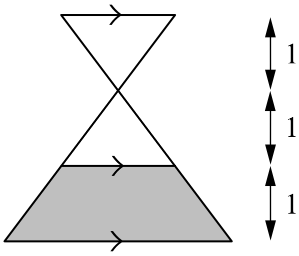


Diagram 4

Diagram 1 shows a triangle with its base divided in the ratio 1 : 3.

Diagram 2 shows a parallelogram with its base divided in the ratio 1 : 3.

Diagram 3 shows a kite with a diagonal divided in the ratio 1 : 3.

Diagram 4 shows two congruent triangles and a trapezium each of height 1 unit.

For each of the four diagrams, write down the **percentage** of the total area which is shaded. [7]

(b)

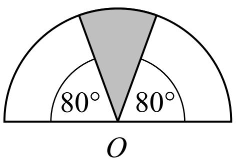


Diagram 5

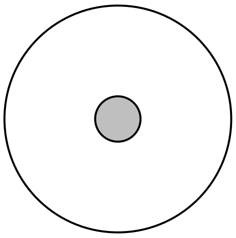


Diagram 6

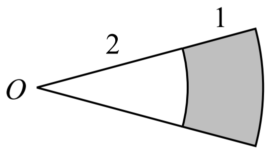


Diagram 7

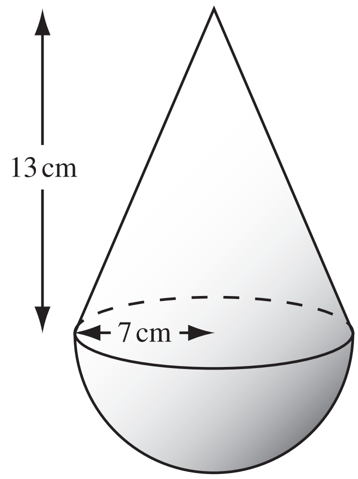
Diagram 5 shows a semicircle, centre O.

Diagram 6 shows two circles with radii 1 unit and 5 units.

Diagram 7 shows two sectors, centre O, with radii 2 units and 3 units.

For each of diagrams 5, 6 and 7, write down the **fraction** of the total area which is shaded. [6]

6



The diagram shows a solid made up of a hemisphere and a cone.
The base radius of the cone and the radius of the hemisphere are each 7 cm.
The height of the cone is 13 cm.

(a) **(i)** Calculate the total volume of the solid.

[The volume of a hemisphere of radius r is given by $V = \frac{2}{3}\pi r^3$.]

[The volume of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.] [2]

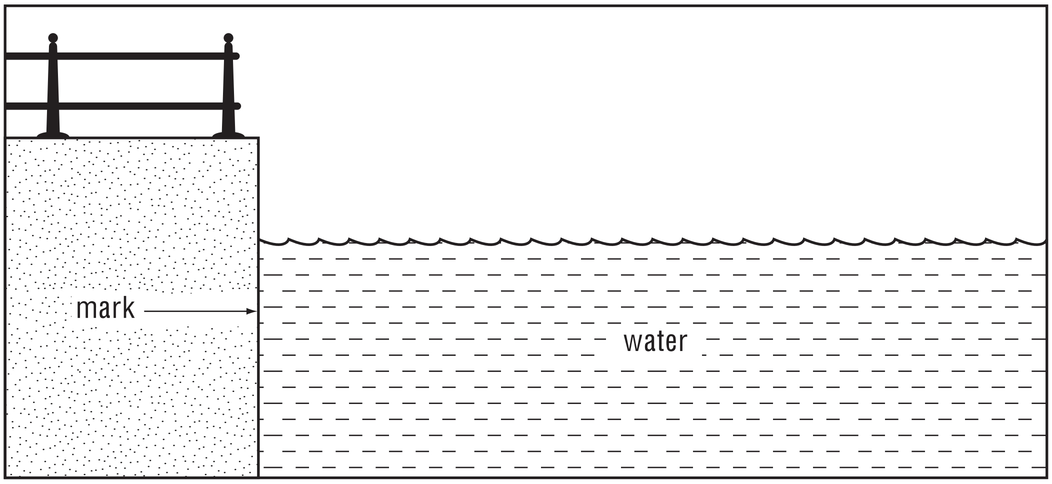
(ii) The solid is made of wood and 1 cm^3 of this wood has a mass of 0.94 g.
Calculate the mass of the solid, in kilograms, correct to 1 decimal place. [3]

(b) Calculate the curved surface area of the cone.
[The curved surface area of a cone of radius r and sloping edge l is given by $A = \pi rl$.] [3]

(c) The cost of covering all the solid with gold plate is $411.58.
Calculate the cost of this gold plate per square centimetre.

[The curved surface area of a **hemisphere** is given by $A = 2\pi r^2$.] [5]

17



The height, h metres, of the water, above a mark on a harbour wall, changes with the tide.
It is given by the equation

$$h = 3\sin(30t)^{\circ}$$

where t is the time in hours after midday.

(a) Calculate the value of h at midday.

Answer (a) .... [1]

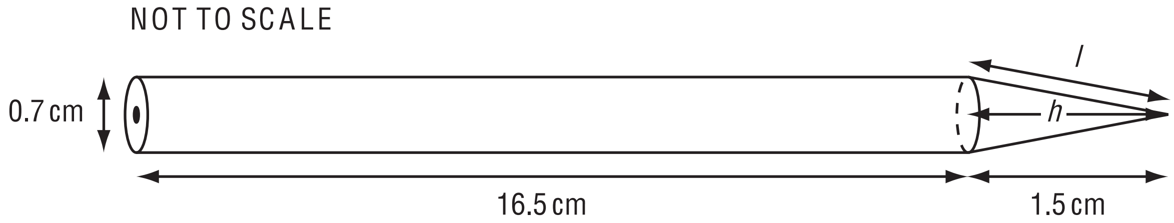
(b) Calculate the value of h at 19 00.

Answer (b) .... [2]

(c) Explain the meaning of the negative sign in your answer.

Answer (c) .... [1]

6

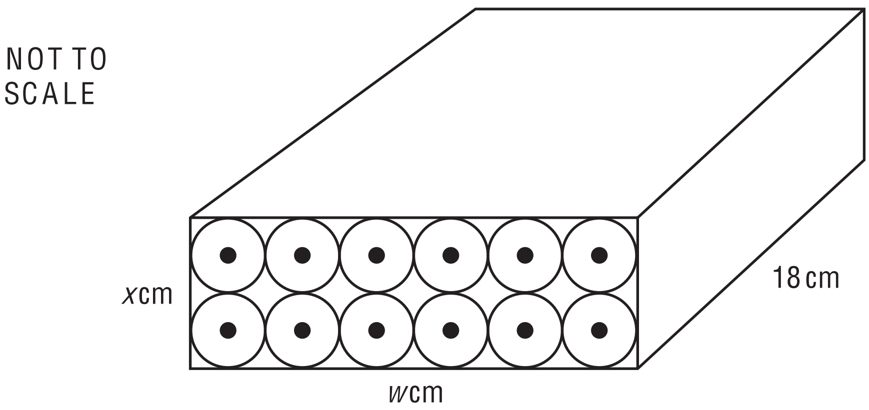


The diagram shows a pencil of length 18 cm.
It is made from a cylinder and a cone.
The cylinder has **diameter** 0.7 cm and length 16.5 cm.
The cone has **diameter** 0.7 cm and length 1.5 cm.

(a) Calculate the volume of the pencil.

[The volume, V, of a cone of radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$.] [3]

(b)



Twelve of these pencils just fit into a rectangular box of length 18 cm, width w cm and height x cm.
The pencils are in 2 rows of 6 as shown in the diagram.

(i) Write down the values of w and x. [2]

(ii) Calculate the volume of the box. [2]

(iii) Calculate the percentage of the volume of the box occupied by the pencils. [2]

(c) Showing all your working, calculate

(i) the slant height, l, of the cone, [2]

(ii) the **total** surface area of **one** pencil, giving your answer correct to 3 significant figures.
[The curved surface area, A, of a cone of radius r and **slant** height l is given by $A = \pi r l$.] [6]

2

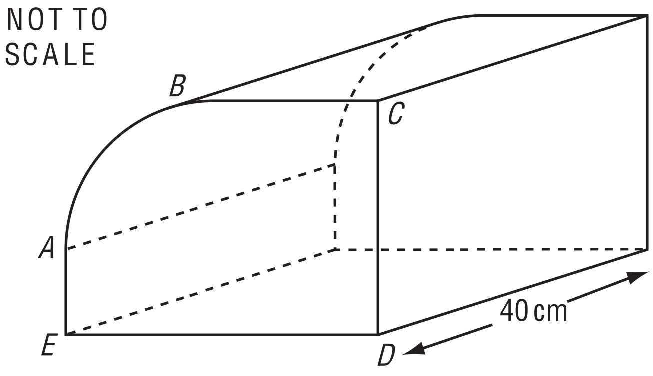


Diagram 1

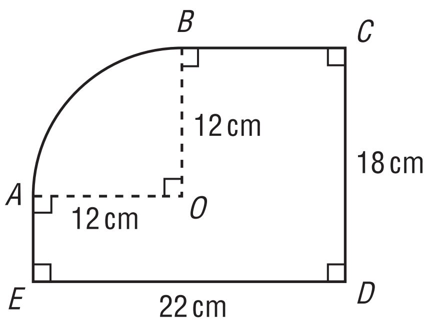


Diagram 2

Diagram 1 shows a closed box. The box is a prism of length 40 cm.
The cross-section of the box is shown in Diagram 2, with all the right-angles marked.
AB is an arc of a circle, centre O, radius 12 cm.
ED = 22 cm and DC = 18 cm.

Calculate

(a) the perimeter of the cross-section, [3]
(b) the area of the cross-section, [3]
(c) the volume of the box, [1]
(d) the **total** surface area of the box. [4]

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3 **Answer the whole of this question on a sheet of graph paper.**

(a) Find the values of k, m and n in each of the following equations, where $a > 0$.

(i) $a^0 = k$, [1]

(ii) $a^m = \dfrac{1}{a}$, [1]

(iii) $a^n = \sqrt{a}^3$. [1]

(b) The table shows some values of the function $f(x) = 2^x$.

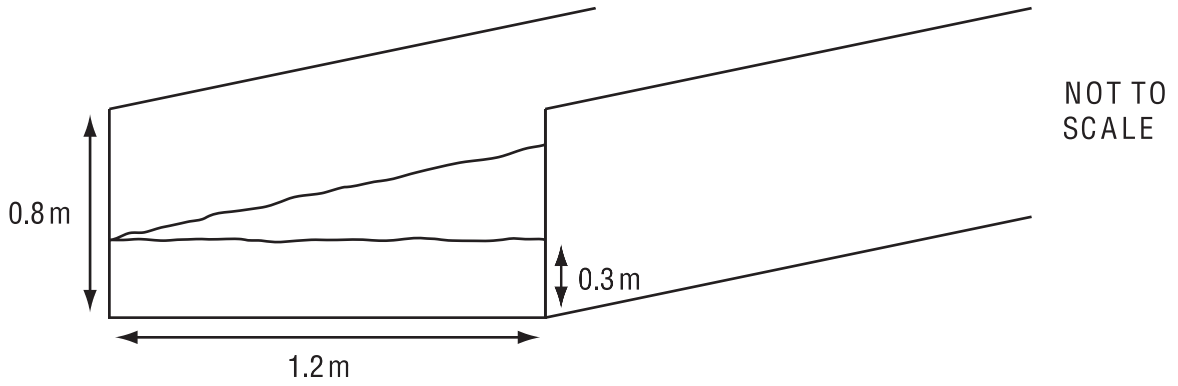
x	-2	-1	-0.5	0	0.5	1	1.5	2	3
$f(x)$	r	0.5	0.71	s	1.41	2	2.83	4	t

(i) Write down the values of r, s and t. [3]
(ii) Using a scale of 2 cm to represent 1 unit on each axis, draw an x-axis from −2 to 3 and a y-axis from 0 to 10. [1]
(iii) On your grid, draw the graph of $y = f(x)$ for $-2 \leqslant x \leqslant 3$. [4]

(c) The function g is given by $g(x) = 6 - 2x$.

(i) On the same grid as **part (b)**, draw the graph of $y = g(x)$ for $-2 \leqslant x \leqslant 3$. [2]
(ii) Use your graphs to solve the equation $2^x = 6 - 2x$. [1]
(iii) Write down the value of x for which $2^x < 6 - 2x$ for $x \in$ {positive integers}. [1]

7



The diagram shows water in a channel.

This channel has a rectangular cross-section, 1.2 metres by 0.8 metres.

(a) When the depth of water is 0.3 metres, the water flows along the channel at 3 metres/**minute**.

Calculate the number of cubic metres which flow along the channel in one hour. [3]

(b) When the depth of water in the channel increases to 0.8 metres, the water flows at 15 metres/minute.

Calculate the percentage increase in the number of cubic metres which flow along the channel in one hour. [4]

(c) The water comes from a cylindrical tank.

When 2 cubic metres of water leave the tank, the level of water in the tank goes down by 1.3 **millimetres**.

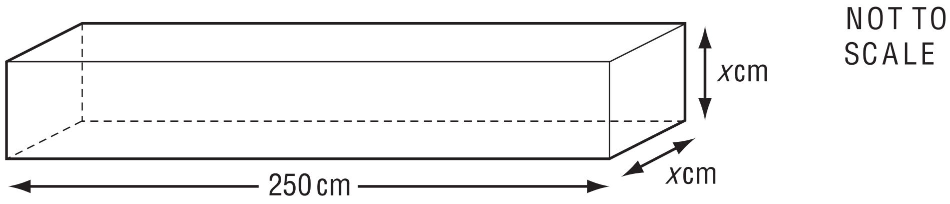
Calculate the radius of the tank, in **metres**, correct to one decimal place. [4]

(d) When the channel is empty, its **interior** surface is repaired.

This costs \$0.12 per square metre. The total cost is \$50.40.

Calculate the length, in metres, of the channel. [4]

7



A solid metal bar is in the shape of a cuboid of length of 250 cm.
The cross-section is a square of side x cm.
The volume of the cuboid is 4840 cm^3.

(a) Show that $x = 4.4$.

Answer (a)

[2]

(b) The mass of 1 cm^3 of the metal is 8.8 grams.
Calculate the mass of the whole metal bar in kilograms.

Answer(b) .... kg [2]

(c) A box, in the shape of a cuboid measures 250 cm by 88 cm by h cm.
120 of the metal bars fit exactly in the box.
Calculate the value of h.

Answer(c) h = ... [2]

(d) One metal bar, of volume 4840 cm^3, is melted down to make 4200 identical small spheres.

All the metal is used.

(i) Calculate the radius of each sphere. Show that your answer rounds to 0.65 cm, correct to 2 decimal places.
[The volume, V, of a sphere, radius r, is given by $V = \frac{4}{3}\pi r^3$.]

Answer(d)(i)

[4]

(ii) Calculate the surface area of each sphere, using 0.65 cm for the radius.
[The surface area, A, of a sphere, radius r, is given by $A = 4\pi r^2$.]

Answer(d)(ii) .... cm^2 [1]

(iii) Calculate the total surface area of all 4200 spheres as a percentage of the surface area of the metal bar.

Answer(d)(iii) .... % [4]

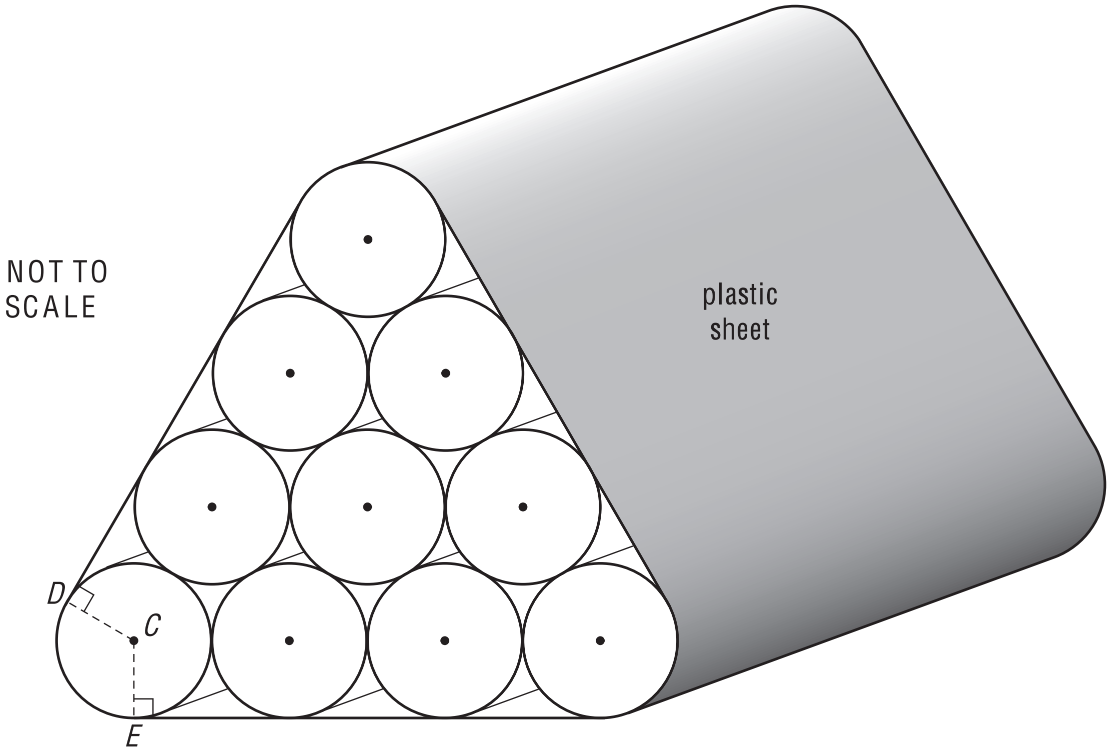
7 **(a)** Calculate the volume of a cylinder of radius 31 **centimetres** and length 15 **metres**.
Give your answer in cubic metres.

Answer(a) .... m^3 [3]

(b) A tree trunk has a circular cross-section of radius 31 cm and length 15 m.
One cubic metre of the wood has a mass of 800 kg.
Calculate the mass of the tree trunk, giving your answer in tonnes.

Answer(b) .... tonnes [2]

(c)



The diagram shows a pile of 10 tree trunks.
Each tree trunk has a circular cross-section of radius 31 cm and length 15 m.
A plastic sheet is wrapped around the pile.

C is the centre of one of the circles.
CE and *CD* are perpendicular to the straight edges, as shown.

(i) Show that angle $ECD = 120°$.

Answer(c)(i)

[2]

(ii) Calculate the length of the arc DE, giving your answer in metres.

Answer(c)(ii) .... m [2]

(iii) The edge of the plastic sheet forms the perimeter of the cross-section of the pile.
The perimeter consists of three straight lines and three arcs.
Calculate this perimeter, giving your answer in metres.

Answer(c)(iii) .... m [3]

(iv) The plastic sheet does not cover the two ends of the pile.
Calculate the area of the plastic sheet.

Answer(c)(iv) .... m^2 [1]

6 A spherical ball has a radius of 2.4 cm.

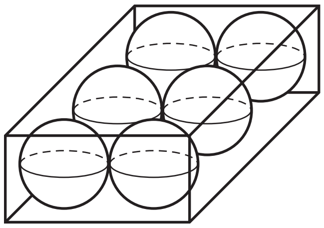
(a) Show that the volume of the ball is 57.9 cm^3, correct to 3 significant figures.

[The volume V of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.]

Answer(a)

[2]

(b)



Six spherical balls of radius 2.4 cm fit exactly into a **closed** box.
The box is a cuboid.

Find

(i) the length, width and height of the box,

Answer(b)(i) .... cm, ... cm, ... cm [3]

(ii) the volume of the box,

Answer(b)(ii) .... cm^3 [1]

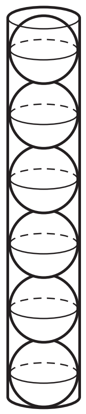
(iii) the volume of the box **not** occupied by the balls,

Answer(b)(iii) .... cm^3 [1]

(iv) the surface area of the box.

Answer(b)(iv) .... cm^2 [2]

(c)



The six balls can also fit exactly into a **closed** cylindrical container, as shown in the diagram.

Find

(i) the volume of the cylindrical container,

Answer(c)(i) .... cm^3 [3]

(ii) the volume of the cylindrical container **not** occupied by the balls,

Answer(c)(ii) .... cm^3 [1]

(iii) the surface area of the cylindrical container.

Answer(c)(iii) .... cm^2 [3]

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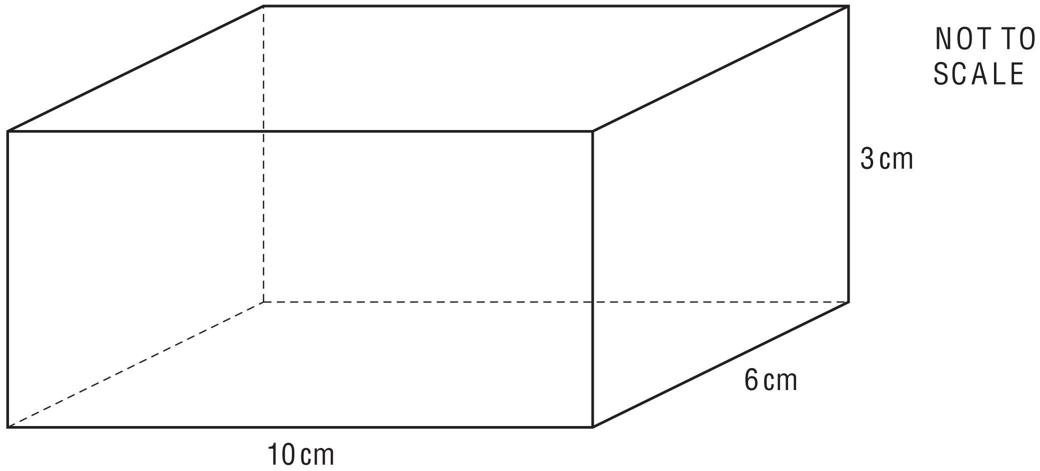


A solid metal cuboid measures 10 cm by 6 cm by 3 cm.

(a) Show that 16 of these solid metal cuboids will fit exactly into a box which has internal measurements 40 cm by 12 cm by 6 cm.

Answer(a)

[2]

(b) Calculate the volume of **one** metal cuboid.

Answer(b) .... cm^3 [1]

(c) One cubic centimetre of the metal has a mass of 8 grams.
The box has a mass of 600 grams.

Calculate the **total** mass of the 16 cuboids **and** the box in

(i) grams,

Answer(c)(i) .... g [2]

(ii) kilograms.

Answer(c)(ii) .... kg [1]

(d) **(i)** Calculate the surface area of **one** of the solid metal cuboids.

Answer(d)(i) .... cm^2 [2]

(ii) The surface of each cuboid is painted. The cost of the paint is \$25 per **square metre**.

Calculate the cost of painting all **16** cuboids.

Answer(d)(ii) \$ .... [3]

(e) **One** of the solid metal cuboids is melted down.
Some of the metal is used to make 200 identical solid spheres of radius 0.5 cm.

Calculate the volume of metal from this cuboid which is **not** used.

[The volume, V, of a sphere of radius r is $V = \frac{4}{3}\pi r^3$.]

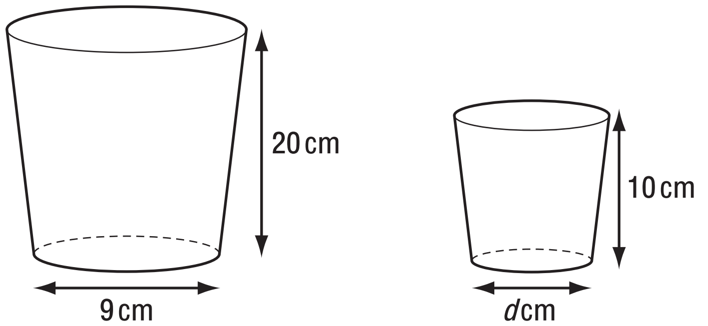
Answer(e) .... cm^3 [3]

(f) 50 cm^3 of metal is used to make 20 identical solid spheres of radius r.

Calculate the radius r.

Answer(f) $r =$ .... cm [3]

17



The diagrams show two mathematically similar containers.
The larger container has a base with diameter 9 cm and a height 20 cm.
The smaller container has a base with diameter *d* cm and a height 10 cm.

(a) Find the value of *d*.

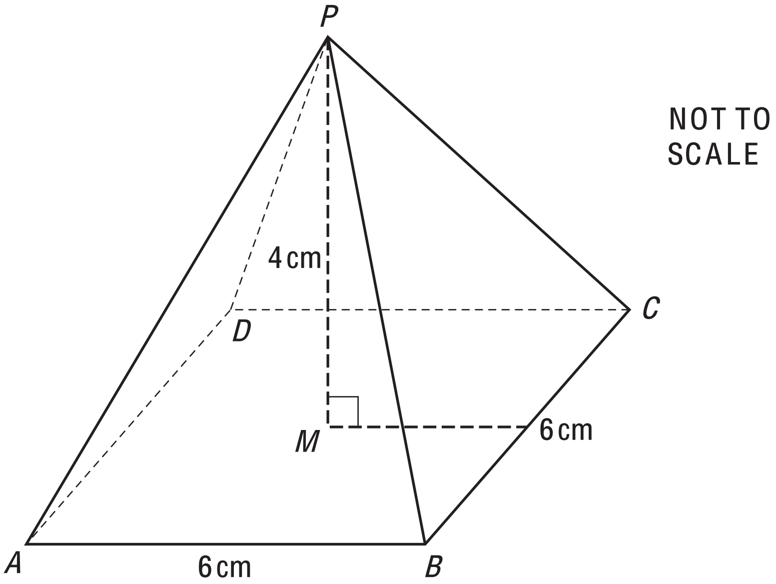
Answer(a) d = ... [1]

(b) The larger container has a capacity of 1600 ml.

Calculate the capacity of the smaller container.

Answer(b) .... ml [2]

21



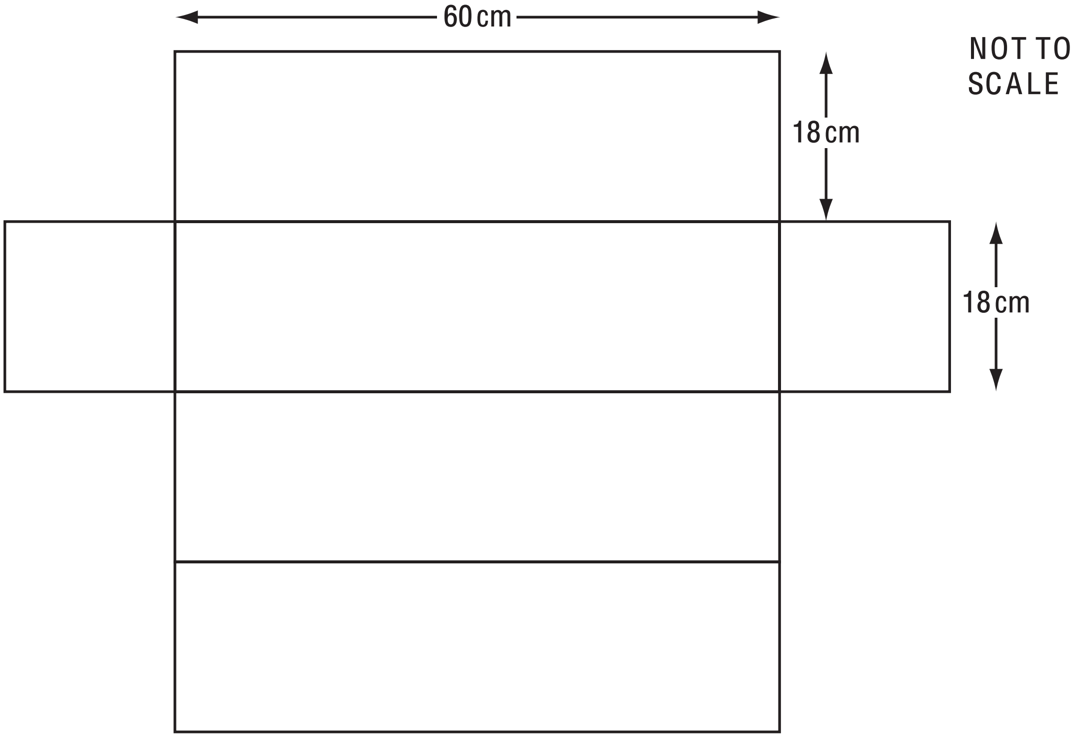
The diagram shows a pyramid with a square base *ABCD* of side 6 cm.

The height of the pyramid, *PM*, is 4 cm, where *M* is the centre of the base.

Calculate the total surface area of the pyramid.

Answer .... cm^2 [5]

9



The diagram shows the net of a box.

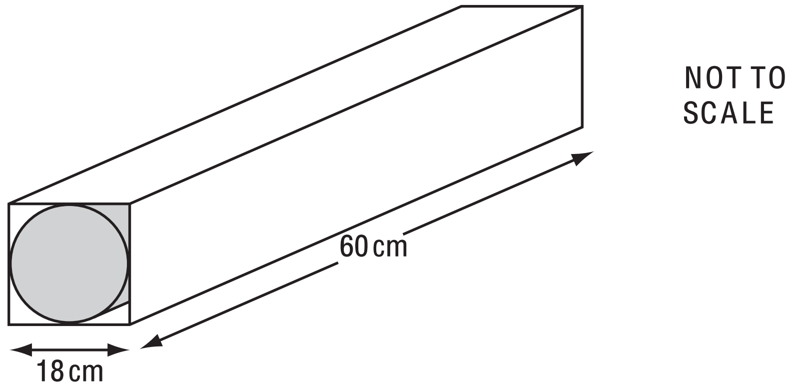
(a) **(i)** Calculate the **total** surface area of the box.

Answer(a)(i) .... cm^2 [2]

(ii) Calculate the volume of the box.

Answer(a)(ii) .... cm^3 [2]

(b) A cylinder with **diameter** 18 cm and length 60 cm just fits inside the box.



(i) Calculate the volume of the cylinder.

Answer(b)(i) .... cm^3 [2]

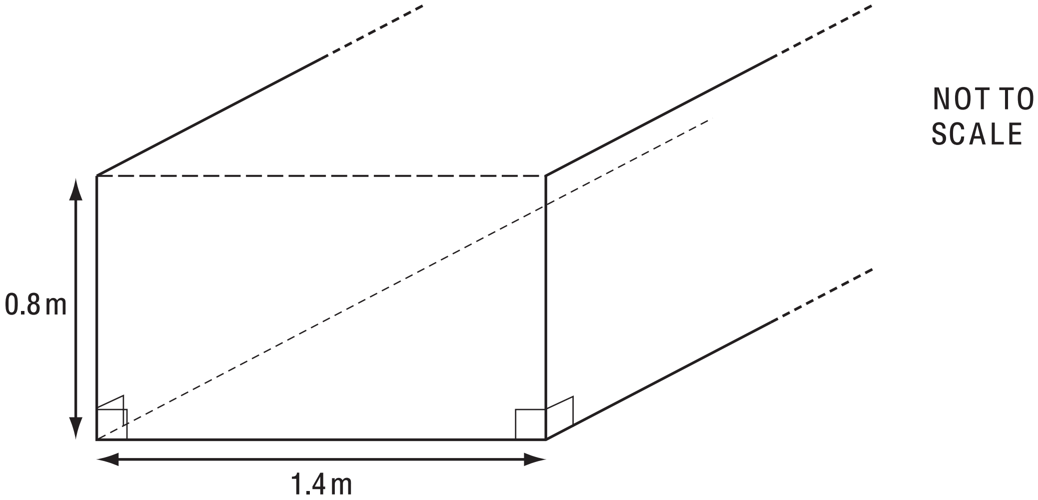
(ii) Find the volume of space outside the cylinder but inside the box.

Answer(b)(ii) .... cm^3 [1]

(iii) Calculate the curved surface area of the cylinder.

Answer(b)(iii) .... cm^2 [2]

4



The diagram shows part of a trench.
The trench is made by removing soil from the ground.
The cross-section of the trench is a rectangle.
The depth of the trench is 0.8 m and the width is 1.4 m.

(a) Calculate the area of the cross-section.

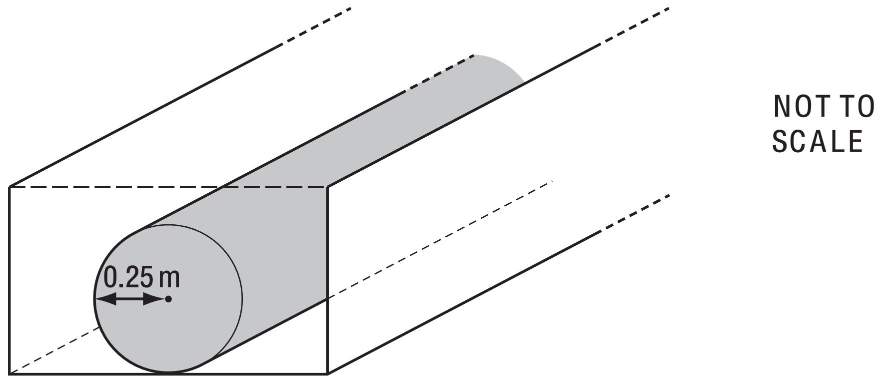
Answer(a) .... m^2 [2]

(b) The length of the trench is 200 m.

Calculate the volume of soil removed.

Answer(b) .... m^3 [1]

(c)



A pipe is put in the trench.
The pipe is a cylinder of radius 0.25 m and length 200 m.

(i) Calculate the volume of the pipe.

[The volume, V, of a cylinder of radius r and length l is $V = \pi r^2 l$.]

Answer(c)(i) .... m^3 [2]

(ii) The trench is then filled with soil.
Find the volume of soil put back into the trench.

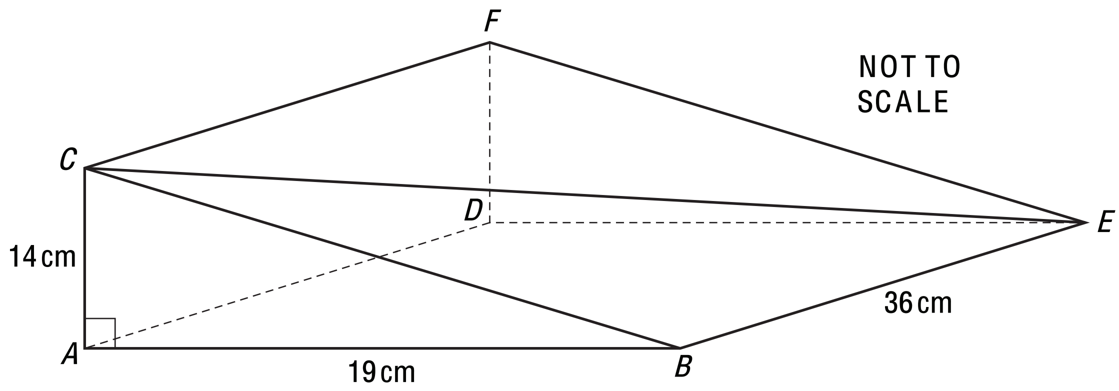
Answer(c)(ii) .... m^3 [1]

(iii) The soil which is **not used** for the trench is spread evenly over a horizontal area of 8000 m^2.

Calculate the depth of this soil.
Give your answer in **millimetres**, correct to 1 decimal place.

Answer(c)(iii) .... mm [3]

6



In the diagram, *ABCDEF* is a prism of length 36 cm.
The cross-section *ABC* is a right-angled triangle.
AB = 19 cm and *AC* = 14 cm.

Calculate

(a) the length *BC*,

Answer(a) BC = ... cm [2]

(b) the total surface area of the prism,

Answer(b) .... cm^2 [4]

(c) the volume of the prism,

Answer(c) .... cm^3 [2]

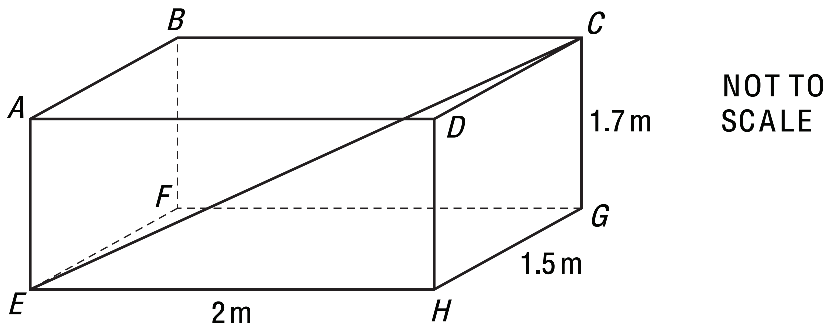
(d) the length *CE*,

Answer(d) CE = ... cm [2]

(e) the angle between the line *CE* and the base *ABED*.

Answer(e) .... [3]

2



The diagram shows a box *ABCDEFGH* in the shape of a cuboid measuring 2 m by 1.5 m by 1.7 m.

(a) Calculate the length of the diagonal *EC*.

Answer(a) EC = ... m [4]

(b) Calculate the angle between *EC* and the base *EFGH*.

Answer(b) .... [3]

(c) **(i)** A rod has length 2.9 m, correct to 1 decimal place.

What is the upper bound for the length of the rod?

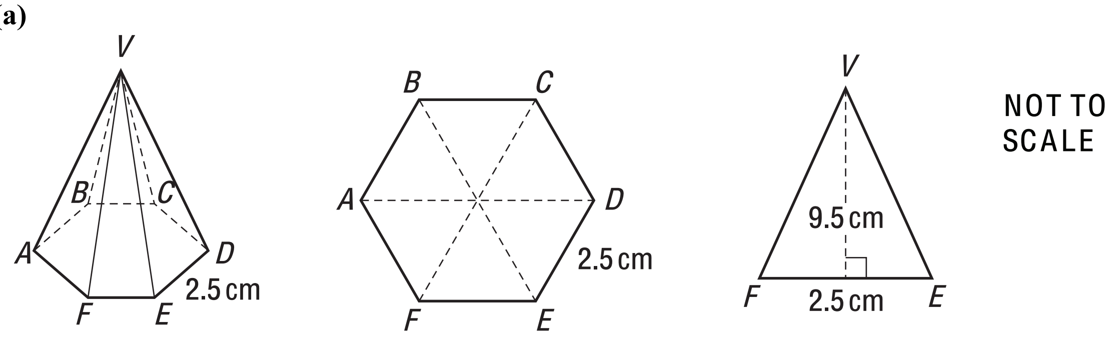
Answer(c)(i) .... m [1]

(ii) Will the rod fit completely in the box?

Give a reason for your answer.

Answer(c)(ii) .... [1]

7 **(a)**

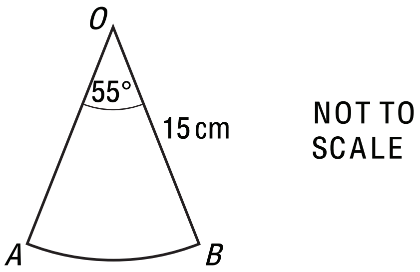


A solid pyramid has a **regular hexagon** of side 2.5 cm as its base.
Each sloping face is an isosceles triangle with base 2.5 cm and height 9.5 cm.

Calculate the **total** surface area of the pyramid.

Answer(a) .... cm^2 [4]

(b)



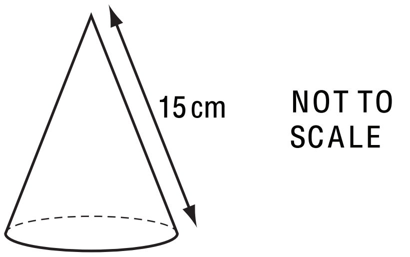
A sector *OAB* has an angle of 55° and a radius of 15 cm.

Calculate the area of the sector and show that it rounds to 108 cm^2, correct to 3 significant figures.

Answer (b)

[3]

(c)



The sector radii *OA* and *OB* in **part (b)** are joined to form a cone.

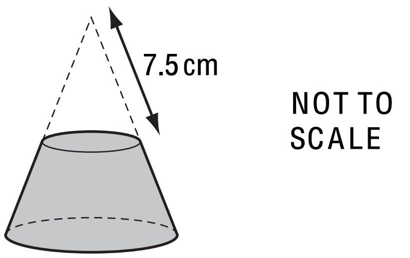
(i) Calculate the base radius of the cone.
[The curved surface area, *A*, of a cone with radius *r* and slant height *l* is $A = \pi rl$.]

Answer(c)(i) .... cm [2]

(ii) Calculate the perpendicular height of the cone.

Answer(c)(ii) .... cm [3]

(d)



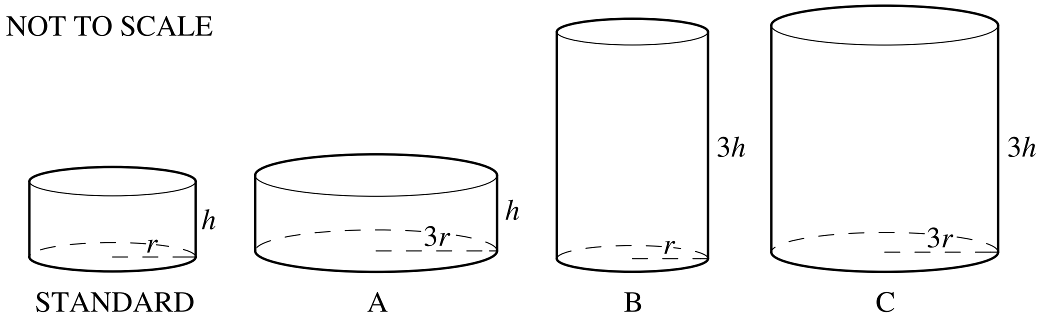
A solid cone has the same dimensions as the cone in **part (c)**.
A small cone with slant height 7.5 cm is removed by cutting parallel to the base.

Calculate the volume of the remaining solid.

[The volume, *V*, of a cone with radius *r* and height *h* is $V = \frac{1}{3}\pi r^2 h$.]

Answer(d) .... cm^3 [3]

8



Sarah investigates cylindrical plant pots.
The standard pot has base radius r cm and height h cm.
Pot A has radius $3r$ and height h. Pot B has radius r and height $3h$. Pot C has radius $3r$ and height $3h$.

(a) **(i)** Write down the volumes of pots A, B and C in terms of π , r and h. [3]

(ii) Find in its lowest terms the ratio of the volumes of $A : B : C$. [2]

(iii) Which one of the pots A, B or C is mathematically similar to the standard pot? Explain your answer. [2]

(iv) The surface area of the standard pot is $S\,\text{cm}^2$. Write down in terms of S the surface area of the similar pot. [2]

(b) Sarah buys a cylindrical plant pot with radius 15 cm and height 20 cm. She wants to paint its outside surface (base and curved surface area).

(i) Calculate the area she wants to paint. [2]

(ii) Sarah buys a tin of paint which will cover $30\,\text{m}^2$.
How many plant pots of this size could be painted on their outside surfaces **completely** using this tin of paint? [4]

---

9 **(a)** Write down the 10th term and the nth term of the following sequences.

(i) 1, 2, 3, 4, 5 …, …, [1]

(ii) 7, 8, 9, 10, 11 …, …, [1]

(iii) 8, 10, 12, 14, 16 …, … . [3]

(b) Consider the sequence

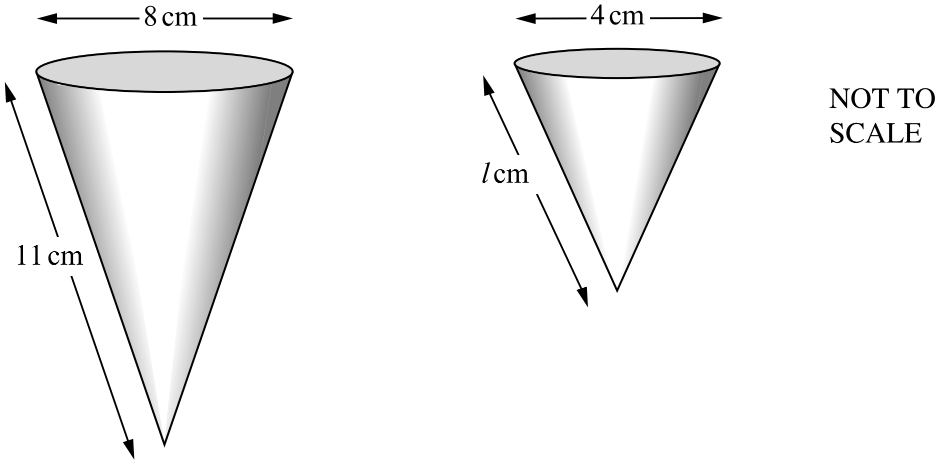
1(8 – 7), 2(10 – 8), 3(12 – 9), 4(14 – 10), …………, ………… .

(i) Write down the next term and the 10th term of this sequence in the form $a(b – c)$ where a, b and c are integers. [3]

(ii) Write down the nth term in the form $a(b – c)$ and then simplify your answer. [2]

For Examiner's use

13



The two cones are similar.

(a) Write down the value of l.

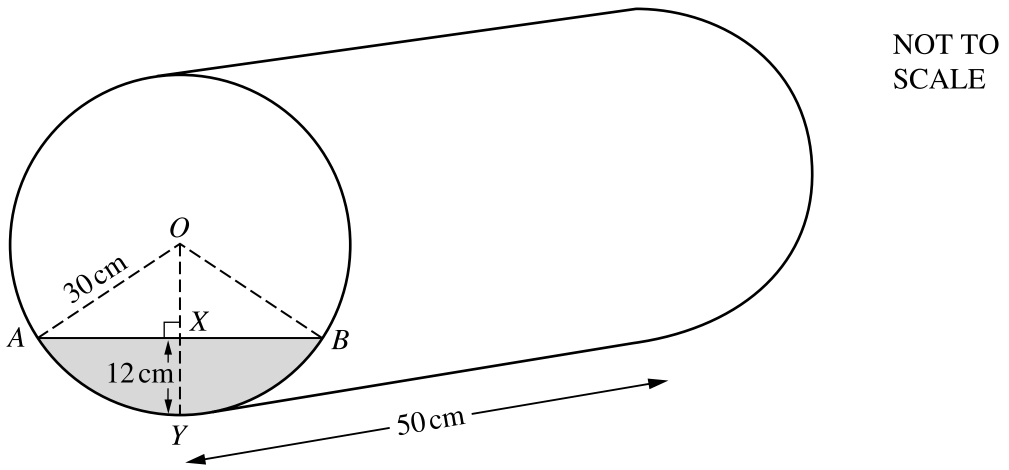
Answer (a) l = ... [1]

(b) When full, the larger cone contains 172 cm^3 of water.
How much water does the smaller cone contain when it is full?

Answer (b) .... cm^3 [2]

6 **(a)** Calculate the volume of a cylinder with radius 30 cm and height 50 cm. [2]

(b)



A cylindrical tank, radius 30 cm and length 50 cm, lies on its side.
It is partially filled with water.
The shaded segment *AXBY* in the diagram shows the cross-section of the water.
The greatest depth, *XY*, is 12 cm.
OA = *OB* = 30 cm.

(i) Write down the length of *OX*. [1]

(ii) Calculate the angle *AOB* correct to two decimal places, showing all your working. [3]

(c) Using angle *AOB* = 106.3°, find

(i) the area of the sector *AOBY*, [3]

(ii) the area of triangle *AOB*, [2]

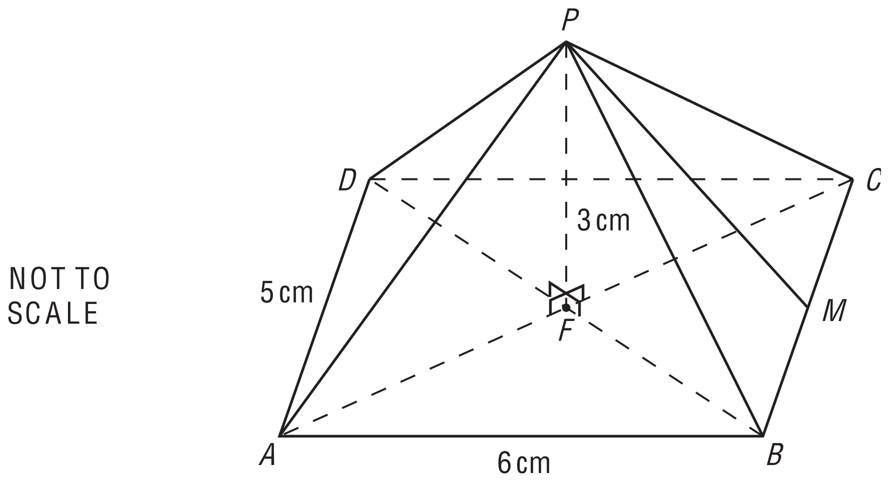
(iii) the area of the shaded segment *AXBY*. [1]

(d) Calculate the volume of water in the cylinder, giving your answer

(i) in cubic centimetres, [2]

(ii) in litres. [1]

(e) How many more litres must be added to make the tank half full? [2]



The diagram shows a pyramid on a rectangular base *ABCD*, with $AB = 6$ cm and $AD = 5$ cm.
The diagonals *AC* and *BD* intersect at *F*.
The vertical height $FP = 3$ cm.

(a) How many planes of symmetry does the pyramid have? [1]

(b) Calculate the volume of the pyramid.

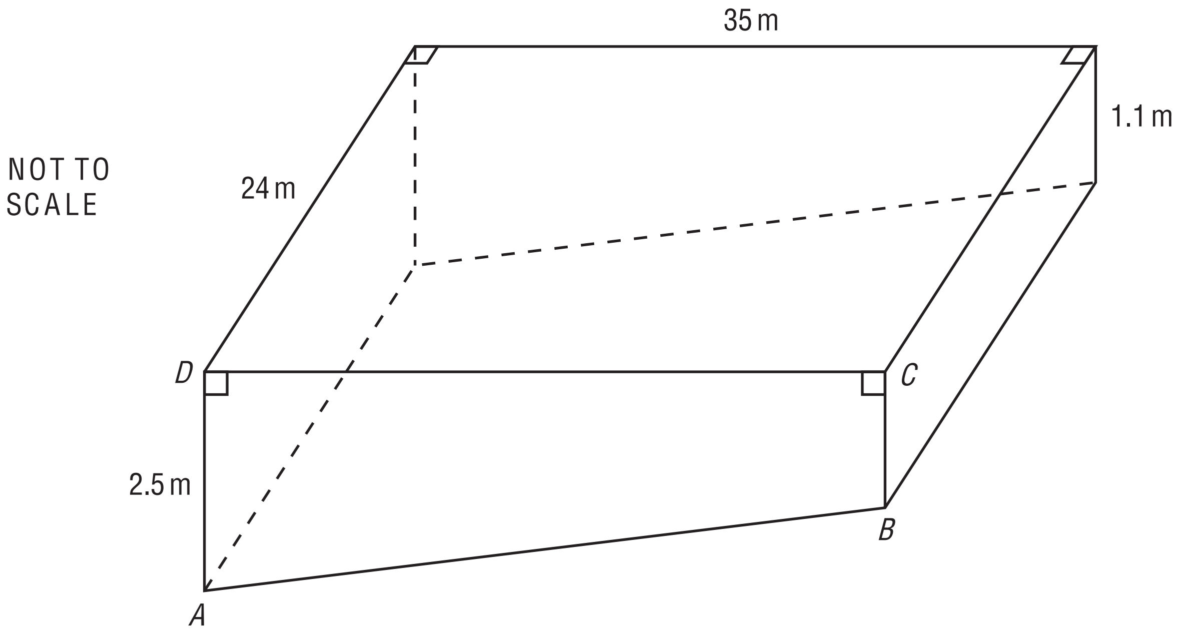
[The volume of a pyramid is $\frac{1}{3}$ × area of base × height.] [2]

(c) The mid-point of *BC* is *M*.
Calculate the angle between *PM* and the base. [2]

(d) Calculate the angle between *PB* and the base. [4]

(e) Calculate the length of *PB*. [2]

7



The diagram shows a swimming pool of length 35 m and width 24 m.
A cross-section of the pool, *ABCD*, is a trapezium with $AD = 2.5$ m and $BC = 1.1$ m.

(a) Calculate

(i) the area of the trapezium *ABCD*, [2]

(ii) the volume of the pool, [2]

(iii) the number of litres of water in the pool, when it is full. [1]

(b) $AB = 35.03$ m correct to 2 decimal places.
The sloping rectangular floor of the pool is painted.
It costs \$2.25 to paint one square metre.

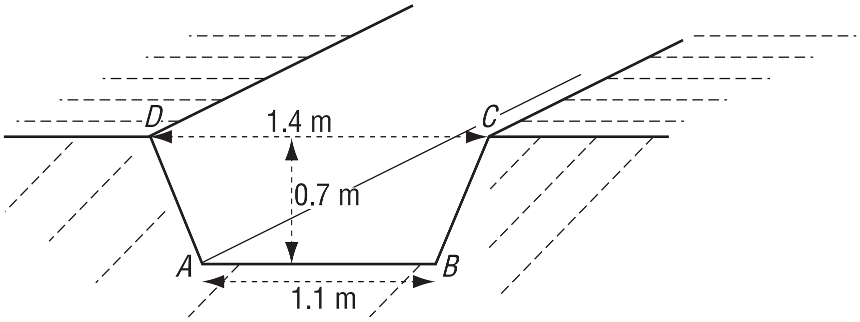
(i) Calculate the cost of painting the floor of the pool. [2]

(ii) Write your answer to **part (b)(i)** correct to the nearest hundred dollars. [1]

(c) **(i)** Calculate the volume of a cylinder, radius 12.5 cm and height 14 cm. [2]

(ii) When the pool is emptied, the water flows through a cylindrical pipe of radius 12.5 cm.
The water flows along this pipe at a rate of 14 centimetres per second.
Calculate the time taken to empty the pool.
Give your answer in days and hours, correct to the nearest hour. [4]

3 Workmen dig a trench in level ground.

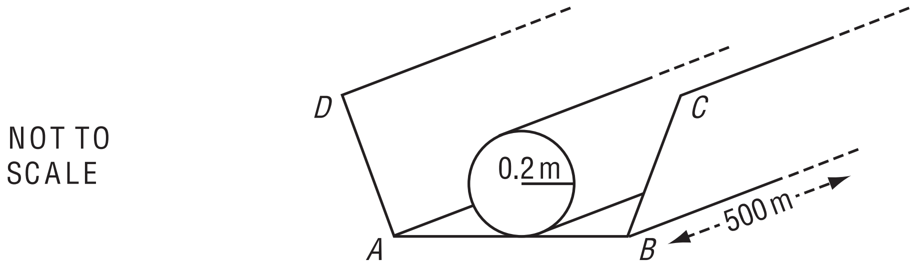


(a) The cross-section of the trench is a trapezium *ABCD* with parallel sides of length 1.1 m and 1.4 m and a vertical height of 0.7 m.
Calculate the area of the trapezium. [2]

(b) The trench is 500 m long.
Calculate the volume of soil removed. [2]

(c) One cubic metre of soil has a mass of 4.8 tonnes.
Calculate the mass of soil removed, giving your answer in tonnes and in standard form. [2]

(d) Change your answer to **part (c)** into grams. [1]



(e) The workmen put a cylindrical pipe, radius 0.2 m and length 500 m, along the bottom of the trench, as shown in the diagram.
Calculate the volume of the cylindrical pipe. [2]

(f) The trench is then refilled with soil.
Calculate the volume of soil put back into the trench as a percentage of the original amount of soil removed. [3]

4 [The surface area of a sphere of radius r is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$.]

(a) A solid metal sphere has a radius of 3.5 cm.

One cubic centimetre of the metal has a mass of 5.6 grams.

Calculate

(i) the surface area of the sphere, [2]

(ii) the volume of the sphere, [2]

(iii) the mass of the sphere. [2]

(b)

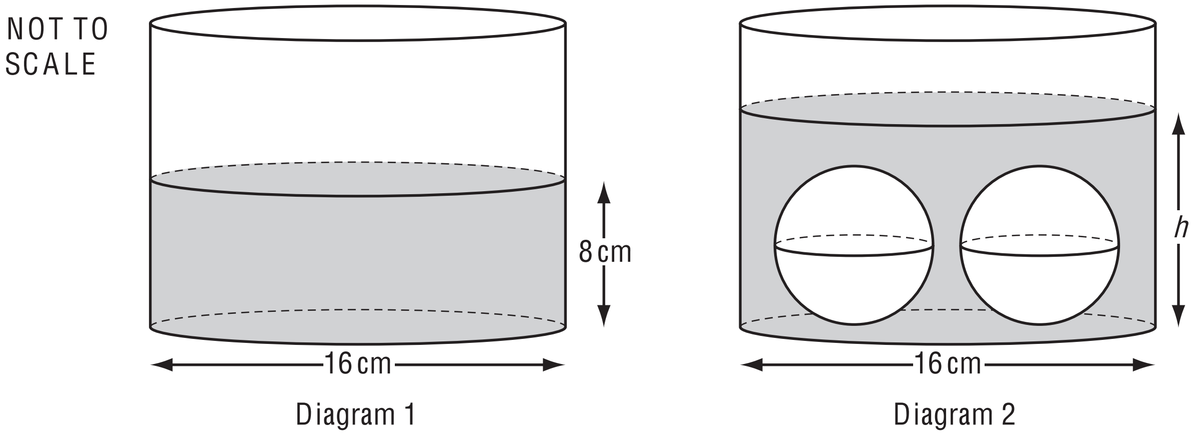


Diagram 1 shows a cylinder with a **diameter** of 16 cm.

It contains water to a depth of 8 cm.

Two spheres identical to the sphere in **part (a)** are placed in the water. This is shown in Diagram 2.

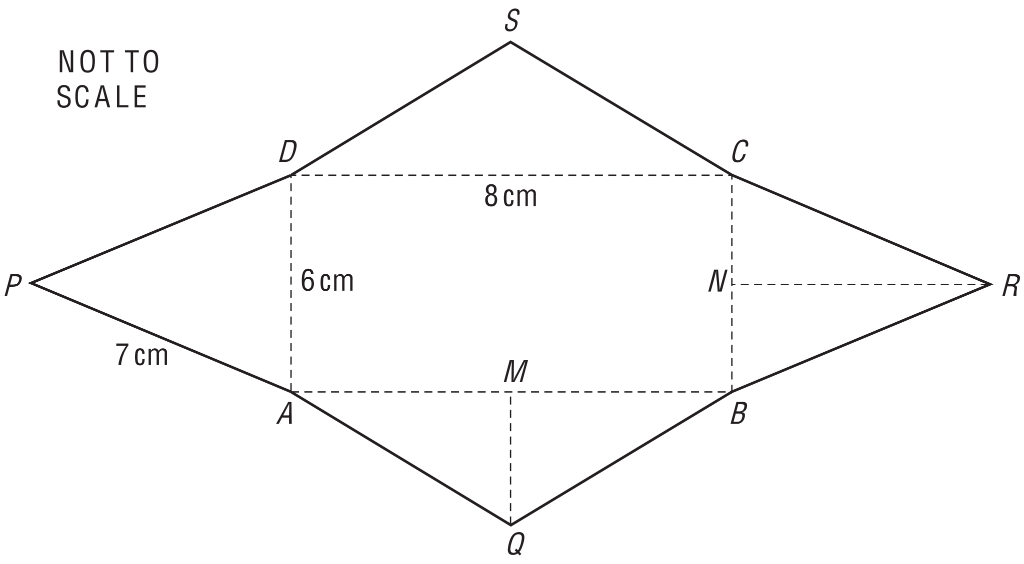
Calculate h, the new depth of water in the cylinder. [4]

(c) A different metal sphere has a mass of 1 kilogram.

One cubic centimetre of this metal has a mass of 4.8 grams.

Calculate the radius of this sphere. [3]

5



The diagram above shows the net of a pyramid.

The base *ABCD* is a rectangle 8 cm by 6 cm.

All the sloping edges of the pyramid are of length 7 cm.

M is the mid-point of *AB* and *N* is the mid-point of *BC*.

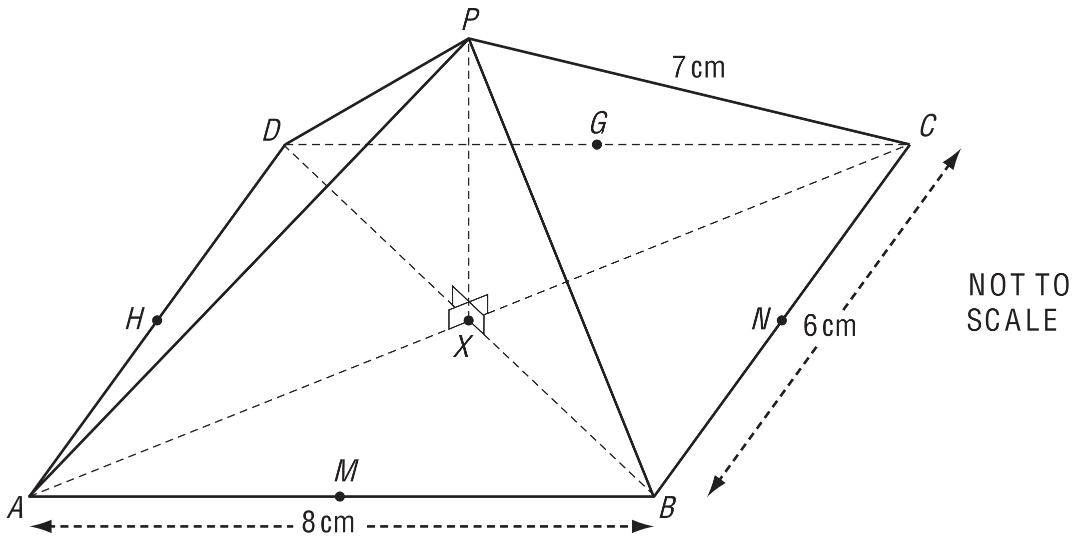
(a) Calculate the length of

(i) *QM*, [2]

(ii) *RN*. [1]

(b) Calculate the surface area of the pyramid. [2]

(c)



The net is made into a pyramid, with *P*, *Q*, *R* and *S* meeting at *P*.

The mid-point of *CD* is *G* and the mid-point of *DA* is *H*.

The diagonals of the rectangle *ABCD* meet at *X*.

(i) Show that the height, *PX*, of the pyramid is 4.90 cm, correct to 2 decimal places. [2]

(ii) Calculate angle *PNX*. [2]

(iii) Calculate angle *HPN*. [2]

(iv) Calculate the angle between the edge *PA* and the base *ABCD*. [3]

(v) Write down the vertices of a triangle which is a plane of symmetry of the pyramid. [1]

4

NOT TO SCALE

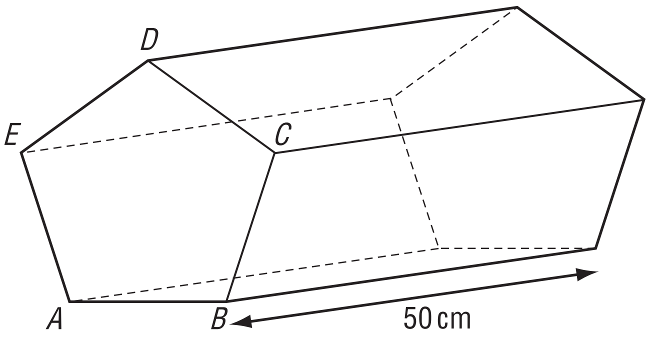


Diagram 1

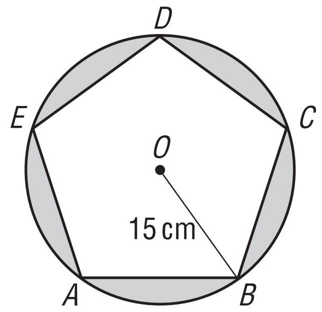


Diagram 2

Diagram 1 shows a solid wooden prism of length 50 cm.

The cross-section of the prism is a regular pentagon *ABCDE*.

The prism is made by removing 5 identical pieces of wood from a solid wooden cylinder.

Diagram 2 shows the cross-section of the cylinder, centre *O,* radius 15 cm.

(a) Find the angle *AOB*. [1]

(b) Calculate

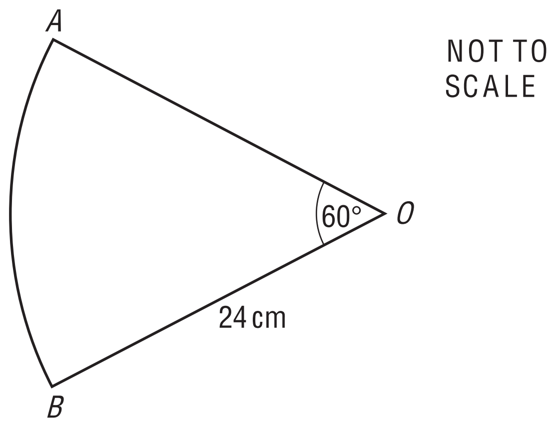
(i) the area of triangle *AOB*, [2]

(ii) the area of the pentagon *ABCDE*, [1]

(iii) the volume of wood removed from the cylinder. [4]

(c) Calculate the total surface area of the prism. [4]

7



(a) The sector of a circle, centre O, radius 24 cm, has angle $AOB = 60°$.

Calculate

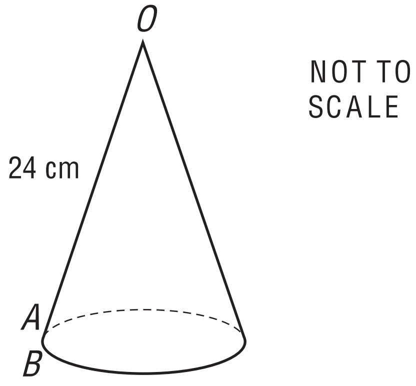
(i) the length of the arc AB,

Answer(a)(i) .... cm [2]

(ii) the area of the sector OAB.

Answer(a)(ii) .... cm^2 [2]

(b) The points A and B of the sector are joined together to make a hollow cone as shown in the diagram. The arc AB of the sector becomes the circumference of the base of the cone.



Calculate

(i) the radius of the base of the cone,

Answer(b)(i) .... cm [2]

(ii) the height of the cone,

Answer(b)(ii) .... cm [2]

(iii) the volume of the cone.
[The volume, V, of a cone of radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(b)(iii) .... cm^3 [2]

(c) A different cone, with radius x and height y, has a volume W.

Find, **in terms of *W***, the volume of

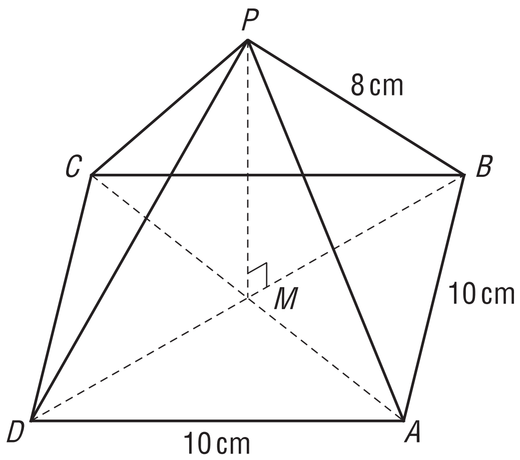
(i) a similar cone, with both radius and height 3 times larger,

Answer(c)(i) .... [1]

(ii) a cone of radius $2x$ and height y.

Answer(c)(ii) .... [1]

19



The diagram represents a pyramid with a square base of side 10 cm.

The diagonals *AC* and *BD* meet at *M*. *P* is vertically above *M* and *PB* = 8cm.

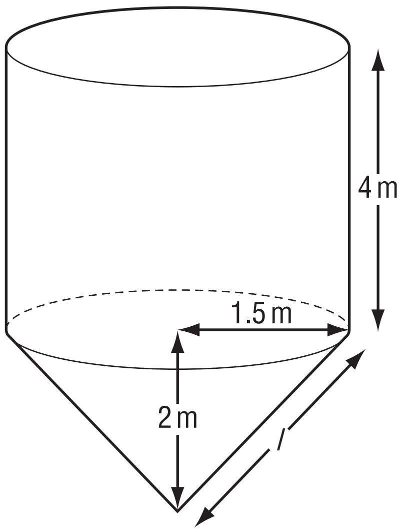
(a) Calculate the length of *BD*.

Answer(a) BD = ... cm [2]

(b) Calculate *MP*, the height of the pyramid.

Answer(b) MP = ... cm [3]

4



An **open** water storage tank is in the shape of a cylinder on top of a cone.
The radius of both the cylinder and the cone is 1.5 m.
The height of the cylinder is 4 m and the height of the cone is 2 m.

(a) Calculate the **total** surface area of the **outside** of the tank.
[The curved surface area, A, of a cone with radius r and slant height l is $A = \pi rl$.]

Answer(a) .... m^2 [6]

(b) The tank is completely full of water.

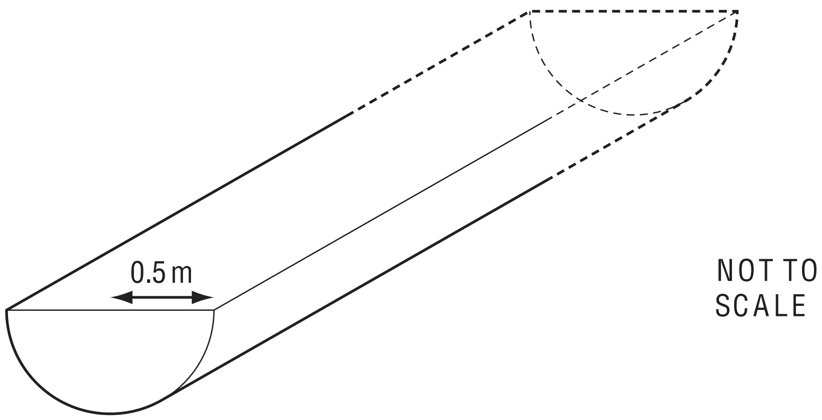
(i) Calculate the volume of water in the tank and show that it rounds to 33 m^3, correct to the nearest whole number.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(b)(i)

[4]

(ii)



The cross-section of an irrigation channel is a semi-circle of radius 0.5 m.
The 33 m^3 of water from the tank completely fills the irrigation channel.

Calculate the length of the channel.

Answer(b)(ii) .... m [3]

(c) (i) Calculate the number of litres in a full tank of 33 m^3.

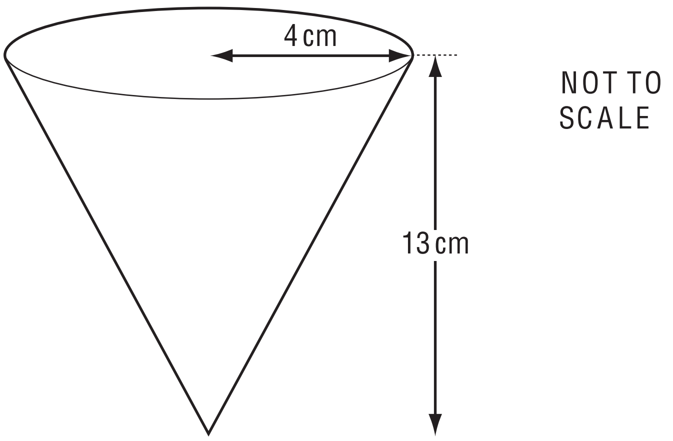
Answer(c)(i) .... litres [1]

(ii) The water drains from the tank at a rate of 1800 litres per minute.

Calculate the time, in minutes and seconds, taken to empty the tank.

Answer(c)(ii) .... min .... s [2]

4 **(a)**



The diagram shows a cone of radius 4 cm and height 13 cm.
It is filled with soil to grow small plants.
Each cubic centimetre of soil has a mass of 2.3g.

(i) Calculate the volume of the soil inside the cone.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]

Answer(a)(i) .... cm^3 [2]

(ii) Calculate the mass of the soil.

Answer(a)(ii) .... g [1]

(iii) Calculate the greatest number of these cones which can be filled **completely** using 50 kg of soil.

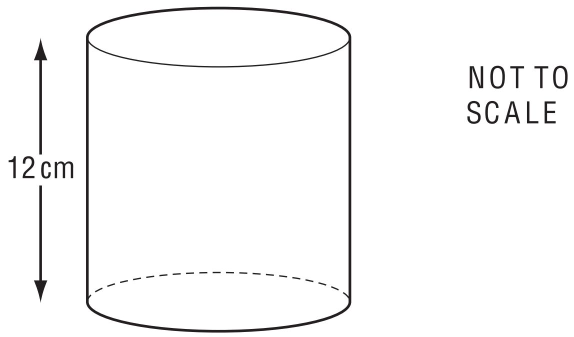
Answer(a)(iii) .... [2]

(b) A **similar** cone of height 32.5 cm is used for growing larger plants.

Calculate the volume of soil used to fill this cone.

Answer(b) .... cm^3 [3]

(c)

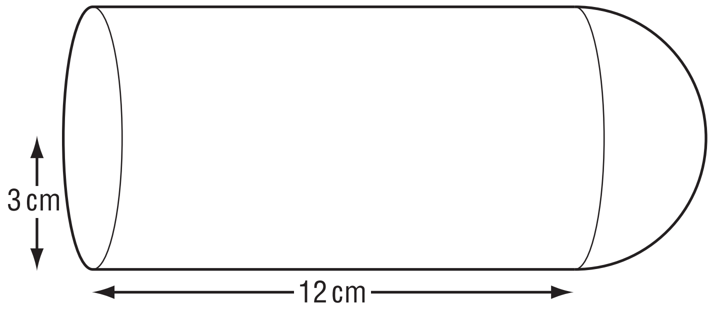


Some plants are put into a cylindrical container with height 12 cm and volume 550 cm^3.

Calculate the radius of the cylinder.

Answer(c) .... cm [3]

8



The diagram shows a solid made up of a hemisphere and a cylinder.
The radius of both the cylinder and the hemisphere is 3 cm.
The length of the cylinder is 12 cm.

(a) **(i)** Calculate the volume of the solid.

[The volume, V, of a **sphere** with radius r is $V = \frac{4}{3}\pi r^3$.]

Answer(a)(i) .... cm^3 [4]

(ii) The solid is made of steel and 1 cm^3 of steel has a mass of 7.9 g.
Calculate the mass of the solid.
Give your answer in kilograms.

Answer(a)(ii) .... kg [2]

(iii) The solid fits into a box in the shape of a cuboid, 15 cm by 6 cm by 6 cm.
Calculate the volume of the box **not** occupied by the solid.

Answer(a)(iii) .... cm^3 [2]

(b) (i) Calculate the **total** surface area of the solid.
You must show your working.
[The surface area, A, of a **sphere** with radius r is $A = 4\pi r^2$.]

Answer(b)(i) .... cm^2 [5]

(ii) The surface of the solid is painted.
The cost of the paint is \$0.09 per millilitre.
One millilitre of paint covers an area of 8 cm^2.
Calculate the cost of painting the solid.

Answer(b)(ii) \$ .... [2]