Sets & Probability
2002 - 2011

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9 **(a)** Emile lost 2 blue buttons from his shirt.

A bag of spare buttons contains 6 white buttons and 2 blue buttons.

Emile takes 3 buttons out of the bag at random **without replacement**.

Calculate the probability that

(i) all 3 buttons are white,

Answer(a)(i) .... [3]

(ii) exactly one of the 3 buttons is blue.

Answer(a)(ii) .... [3]

9 **(a)** Emile lost 2 blue buttons from his shirt.

A bag of spare buttons contains 6 white buttons and 2 blue buttons.

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(i) all 3 buttons are white,

Answer(a)(i) .... [3]

(ii) exactly one of the 3 buttons is blue.

Answer(a)(ii) .... [3]

(b) There are 25 buttons in another bag.

This bag contains x blue buttons.

Two buttons are taken at random **without replacement**.

The probability that they are both blue is $\frac{7}{100}$.

9

Set A $\boxed{S}\ \boxed{U}\ \boxed{M}\ \boxed{S}$

Set B $\boxed{M}\ \boxed{I}\ \boxed{N}\ \boxed{U}\ \boxed{S}$

The diagram shows two sets of cards.

(a) One card is chosen at random from Set A and replaced.

(i) Write down the probability that the card chosen shows the letter M.

Answer(a)(i) .... [1]

(ii) If this is carried out 100 times, write down the expected number of times the card chosen shows the letter M.

Answer(a)(ii) .... [1]

(b) Two cards are chosen at random, **without** replacement, from Set A.

Find the probability that both cards show the letter S.

Answer(b) .... [2]

(c) One card is chosen at random from Set A and one card is chosen at random from Set B.

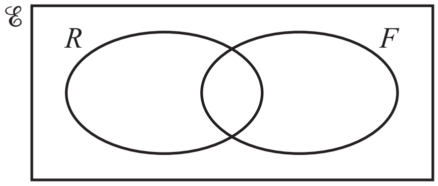
Find the probability that exactly one of the two cards shows the letter U.

Answer(c) .... [3]

(d) A card is chosen at random, **without** replacement, from Set B until the letter shown is either I or U.

Find the probability that this does not happen until the 4th card is chosen.

17



In the Venn diagram, ℰ = {students in a survey}, R = {students who like rugby} and F = {students who like football}.

n(ℰ) = 20 $\quad$ $n(R \cup F) = 17$ $\quad$ $n(R) = 13$ $\quad$ $n(F) = 11$

(a) Find

(i) $n(R \cap F)$,

Answer(a)(i) .... [1]

(ii) $n(R' \cap F)$.

Answer(a)(ii) .... [1]

(b) A student who likes rugby is chosen at random.

Find the probability that this student also likes football.

Answer(b) .... [1]

For Examiner's Use

16 In a survey of 60 cars, the type of fuel that they use is recorded in the table below.

Each car only uses one type of fuel.

Petrol	Diesel	Liquid Hydrogen	Electricity
40	12	2	6

(a) Write down the mode.

Answer(a) .... [1]

(b) Olav drew a pie chart to illustrate these figures.

Calculate the angle of the sector for Diesel.

Answer(b) .... [2]

(c) Calculate the probability that a car chosen at random uses Electricity.

Write your answer as a fraction in its simplest form.

Answer(c) .... [2]

3

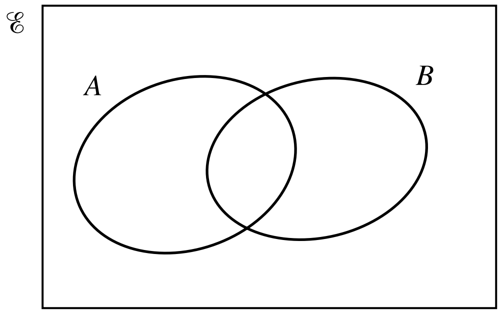


Diagram 1

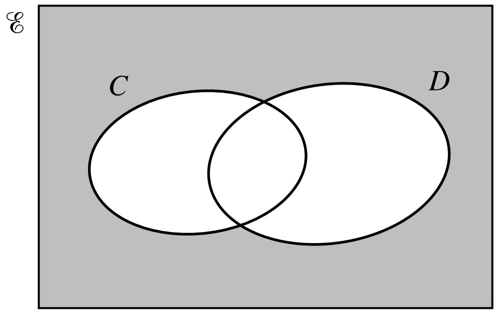


Diagram 2

(a) In Diagram 1, shade the area which represents $A \cup B'$. [1]

(b) Describe in set notation the shaded area in Diagram 2.

Answer (b) .... [1]

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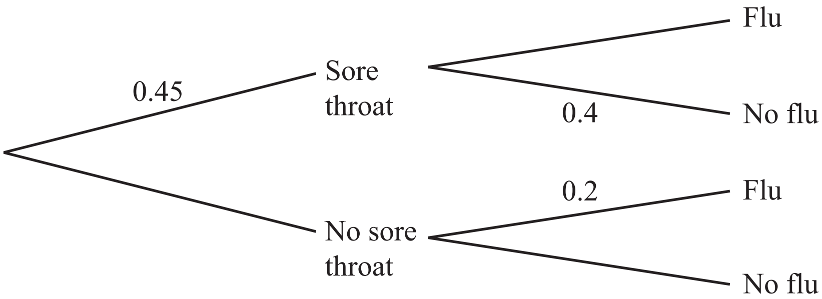
10 In a flu epidemic 45% of people have a sore throat.
If a person has a sore throat the probability of **not** having flu is 0.4.
If a person does not have a sore throat the probability of having flu is 0.2.

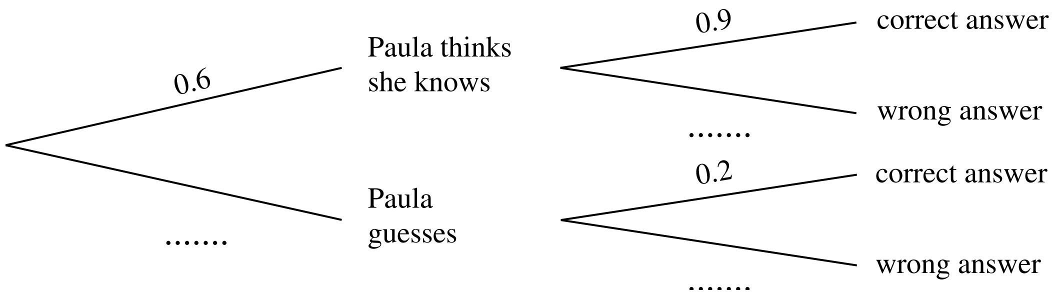


Calculate the probability that a person chosen at random has flu.

Answer .... [4]

3 Paula and Tarek take part in a quiz.
The probability that Paula thinks she knows the answer to any question is 0.6.
If Paula thinks she knows, the probability that she is correct is 0.9.
Otherwise she guesses and the probability that she is correct is 0.2.

(a) **Copy and complete** the tree diagram.



[3]

(b) Find the probability that Paula

(i) thinks she knows the answer and is correct, [1]

(ii) gets the correct answer. [2]

(c) The probability that Tarek thinks he knows the answer to any question is 0.55.
If Tarek thinks he knows, he is always correct.
Otherwise he guesses and the probability that he is correct is 0.2.

(i) Draw a tree diagram for Tarek. Write all the probabilities on your diagram. [3]

(ii) Find the probability that Tarek gets the correct answer. [2]

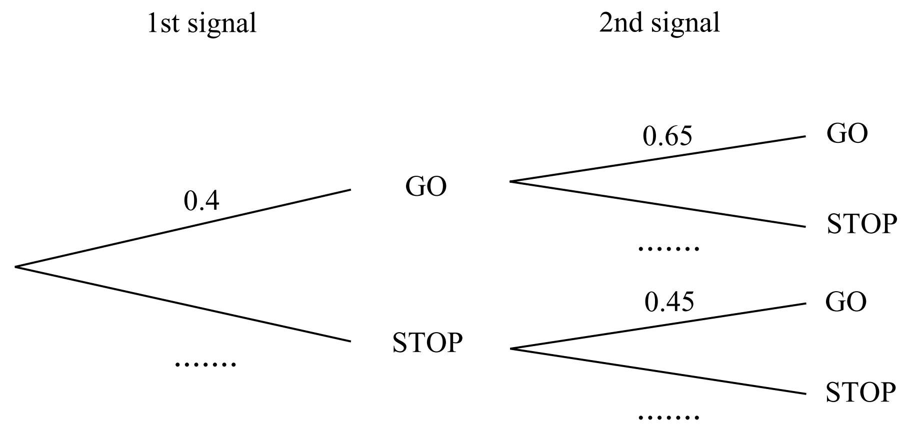
(d) There are 100 questions in the quiz.
Estimate the number of correct answers given by

(i) Paula, [1]

(ii) Tarek. [1]

3 There are 2 sets of road signals on the direct 12 kilometre route from Acity to Beetown.
The signals say either "GO" or "STOP".
The probabilities that the signals are "GO" when a car arrives are shown in the tree diagram.

(a) **Copy and complete** the tree diagram for a car driver travelling along this route.



[3]

(b) Find the probability that a car driver

(i) finds both signals are "GO", [2]

(ii) finds exactly one of the two signals is "GO", [3]

(iii) does not find two "STOP" signals. [2]

(c) With no stops, Damon completes the 12 kilometre journey at an average speed of 40 kilometres per hour.

(i) Find the time taken in **minutes** for this journey. [1]

(ii) When Damon has to stop at a signal it adds 3 minutes to this journey time.

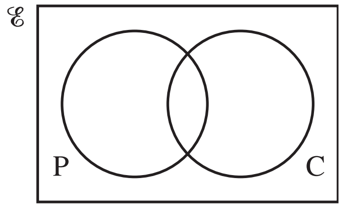
Calculate his average speed, in **kilometres per hour**, if he stops at both road signals. [2]

(d) Elsa takes a different route from Acity to Beetown.
This route is 15 kilometres and there are no road signals.
Elsa's average speed for this journey is 40 kilometres per hour.
Find

(i) the time taken in **minutes** for this journey, [1]

(ii) the probability that Damon takes more time than this on his 12 kilometre journey. [2]

7 (a) There are 30 students in a class.
20 study Physics, 15 study Chemistry and 3 study neither Physics nor Chemistry.



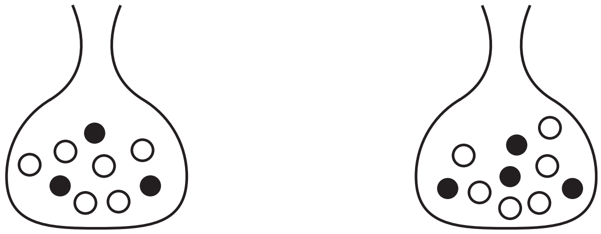
(i) **Copy and complete** the Venn diagram to show this information. [2]

(ii) Find the number of students who study both Physics **and** Chemistry. [1]

(iii) A student is chosen at random. Find the probability that the student studies Physics but not Chemistry. [2]

(iv) A student who studies Physics is chosen at random. Find the probability that this student does not study Chemistry. [2]

(b)



Bag A contains 6 white beads and 3 black beads.
Bag B contains 6 white beads and 4 black beads.
One bead is chosen at random from each bag.
Find the probability that

(i) both beads are black, [2]

(ii) at least one of the two beads is white. [2]

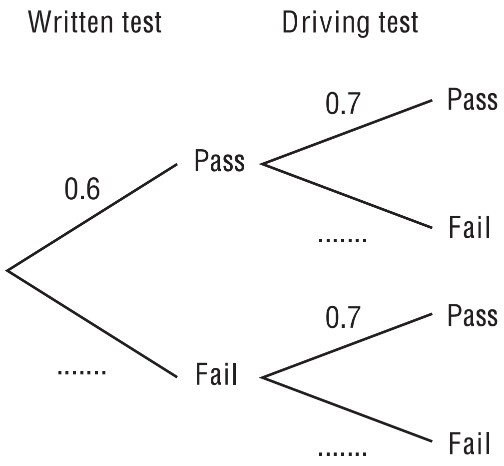
The beads are not replaced.
A second bead is chosen at random from each bag.
Find the probability that

(iii) all four beads are white, [3]

(iv) the beads are not all the same colour. [3]

18 Revina has to pass a written test and a driving test before she can drive a car on her own.
The probability that she passes the written test is 0.6.
The probability that she passes the driving test is 0.7.

(a) Complete the tree diagram below.



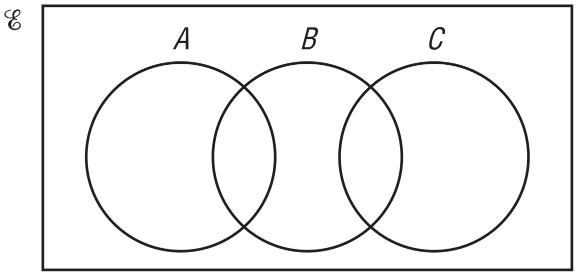
[1]

(b) Calculate the probability that Revina passes only one of the two tests.

Answer (b) .... [3]

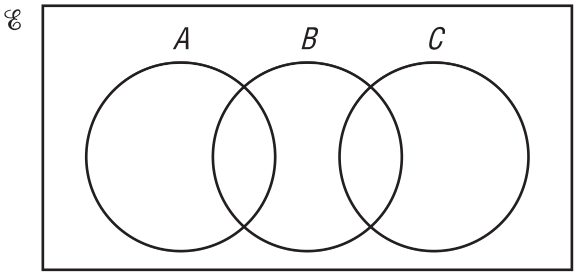
8 On the Venn diagrams shade the regions

(a) $A' \cap C'$,



[1]

(b) $(A \cup C) \cap B$.



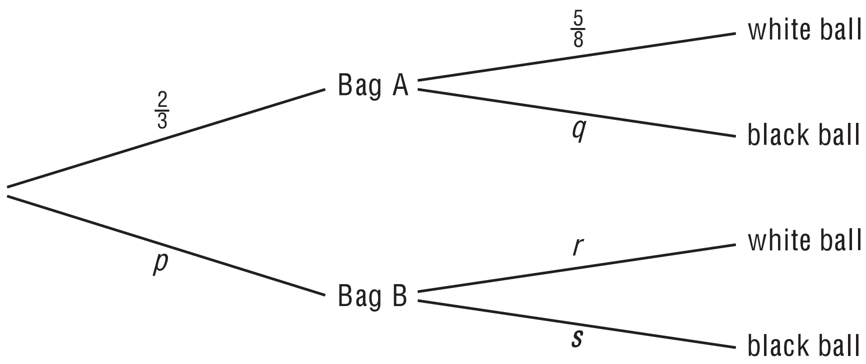
[1]

3 **(a)**

Nadia must choose a ball from Bag A or from Bag B.
The probability that she chooses Bag A is $\frac{2}{3}$.

Bag A contains 5 white and 3 black balls.
Bag B contains 6 white and 2 black balls.

The tree diagram below shows some of this information.



(i) Find the values of p, q, r and s. [3]

(ii) Find the probability that Nadia chooses Bag A and then a white ball. [2]

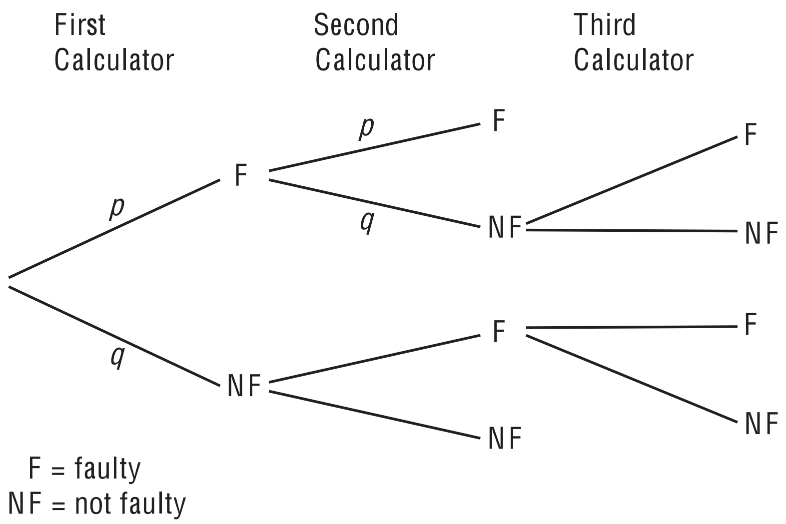
(iii) Find the probability that Nadia chooses a white ball. [2]

(b) Another bag contains 7 green balls and 3 yellow balls.
Sani takes three balls out of the bag, without replacement.

(i) Find the probability that all three balls he chooses are yellow. [2]

(ii) Find the probability that at least one of the three balls he chooses is green. [1]

8



The tree diagram shows a testing procedure on calculators, taken from a large batch.

Each time a calculator is chosen at random, the probability that it is faulty (F) is $\frac{1}{20}$.

(a) Write down the values of p and q.

Answer(a) p = ... and q = ... [1]

(b) Two calculators are chosen at random.

Calculate the probability that

(i) both are faulty,

Answer(b)(i) .... [2]

(ii) **exactly one** is faulty.

Answer(b)(ii) .... [2]

(c) If **exactly one** out of two calculators tested is faulty, then a third calculator is chosen at random.

Calculate the probability that exactly one of the first two calculators is faulty **and** the third one is faulty.

Answer(c) .... [2]

(d) The whole batch of calculators is rejected

either if the first two chosen are both faulty

or if a third one needs to be chosen and it is faulty.

Calculate the probability that the whole batch is rejected.

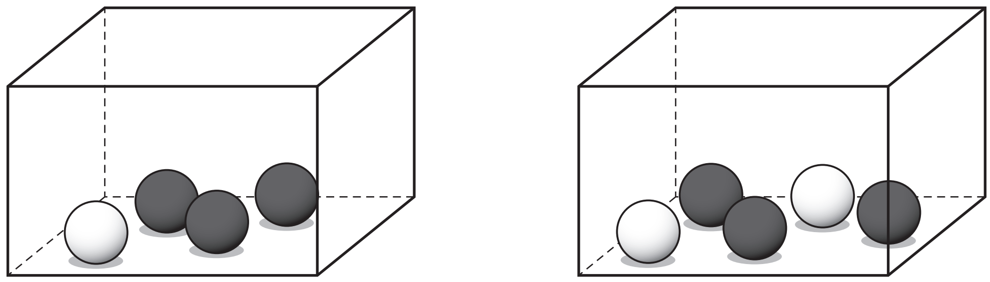
Answer(d) .... [2]

(e) In one month, 1000 batches of calculators are tested in this way.

How many batches are expected to be rejected?

Answer(e) .... [1]

4



A B

Box A contains 3 black balls and 1 white ball.
Box B contains 3 black balls and 2 white balls.

(a) A ball can be chosen at random from either box.
Complete the following statement.

There is a greater probability of choosing a white ball from Box ....

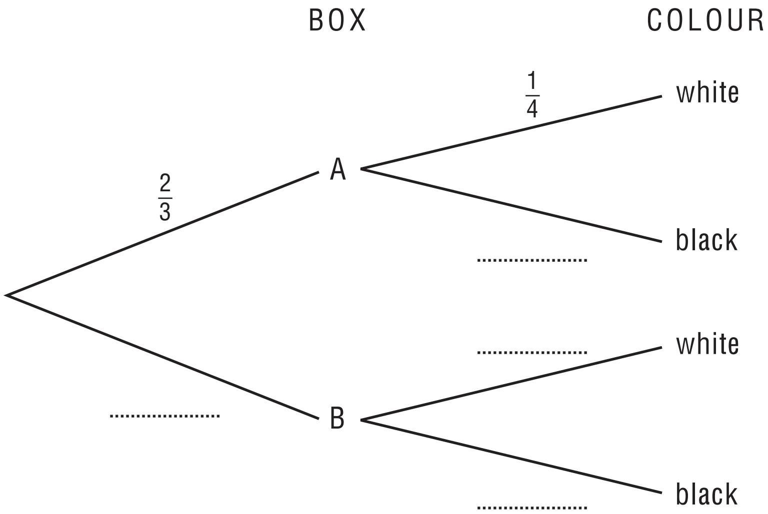
Explain your answer.

Answer(a) .... [1]

(b) Abdul chooses a box and then chooses a ball from this box at random.

The probability that he chooses box A is $\frac{2}{3}$.

(i) Complete the tree diagram by writing the four probabilities in the empty spaces.



[4]

(ii) Find the probability that Abdul chooses box A and a black ball.

Answer(b)(ii) .... [2]

(iii) Find the probability that Abdul chooses a black ball.

Answer(b)(iii) .... [2]

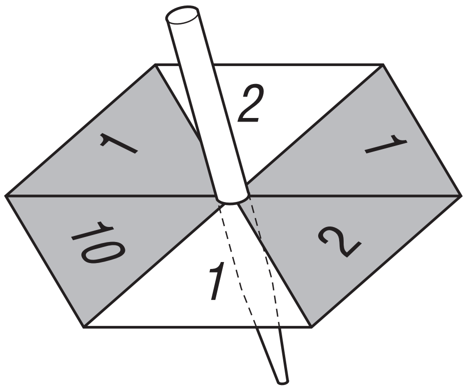
(c) Tatiana chooses a box and then chooses **two** balls from this box at random (without replacement).

The probability that she chooses box A is $\frac{2}{3}$.

Find the probability that Tatiana chooses two white balls.

Answer(c) .... [2]

3



The diagram shows a spinner with six numbered sections.
Some of the sections are shaded.
Each time the spinner is spun it stops on one of the six sections.
It is equally likely that it stops on any one of the sections.

(a) The spinner is spun once.

Find the probability that it stops on

(i) a shaded section,

Answer(a)(i) .... [1]

(ii) a section numbered 1,

Answer(a)(ii) .... [1]

(iii) a shaded section numbered 1,

Answer(a)(iii) .... [1]

(iv) a shaded section or a section numbered 1.

Answer(a)(iv) .... [1]

(b) The spinner is now spun twice.

Find the probability that the total of the two numbers is

(i) 20,

Answer(b)(i) .... [2]

(ii) 11.

Answer(b)(ii) .... [2]

(c) **(i)** The spinner stops on a shaded section.

Find the probability that this section is numbered 2.

Answer(c)(i) .... [1]

(ii) The spinner stops on a section numbered 2.

Find the probability that this section is shaded.

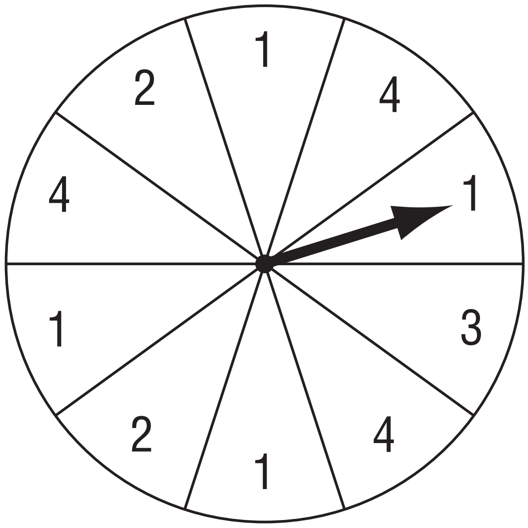
Answer(c)(ii) .... [1]

(d) The spinner is now spun until it stops on a section numbered 2.
The probability that this happens on the nth spin is $\frac{16}{243}$.

Find the value of n.

Answer(d) n = ... [2]

3



The diagram shows a circular board, divided into 10 numbered sectors.

When the arrow is spun it is equally likely to stop in any sector.

(a) Complete the table below which shows the probability of the arrow stopping at each number.

Number	1	2	3	4
Probability		0.2		0.3

[1]

(b) The arrow is spun once.

Find

(i) the most likely number,

Answer(b)(i) .... [1]

(ii) the probability of a number less than 4.

Answer(b)(ii) .... [1]

(c) The arrow is spun twice.

Find the probability that

(i) both numbers are 2,

Answer(c)(i) .... [1]

(ii) the first number is 3 and the second number is 4,

Answer(c)(ii) .... [2]

(iii) the two numbers add up to 4.

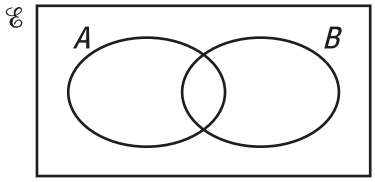
Answer(c)(iii) .... [3]

(d) The arrow is spun several times until it stops at a number 4.

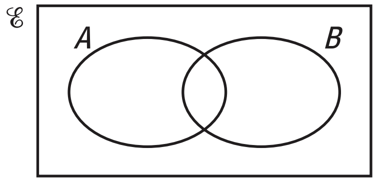
Find the probability that this happens on the third spin.

Answer(d) .... [2]

2 Shade the required region on each Venn diagram.



$A \cup B'$



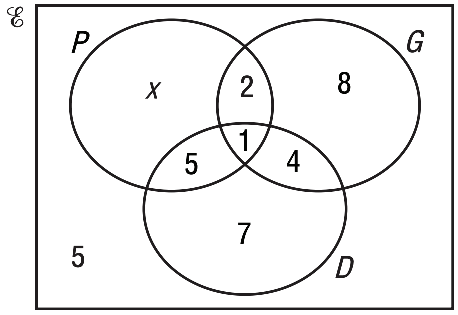
$(A \cap B)'$

[2]

15 A teacher asks 36 students which musical instruments they play.

P = {students who play the piano}
G = {students who play the guitar}
D = {students who play the drums}

The Venn diagram shows the results.



(a) Find the value of x.

Answer(a) x = ... [1]

(b) A student is chosen at random.

Find the probability that this student

(i) plays the drums but **not** the guitar,

Answer(b)(i) .... [1]

(ii) plays only 2 different instruments.

Answer(b)(ii) .... [1]

(c) A student is chosen at random from those who play the guitar.

Find the probability that this student plays no other instrument.

Answer(c) .... [1]

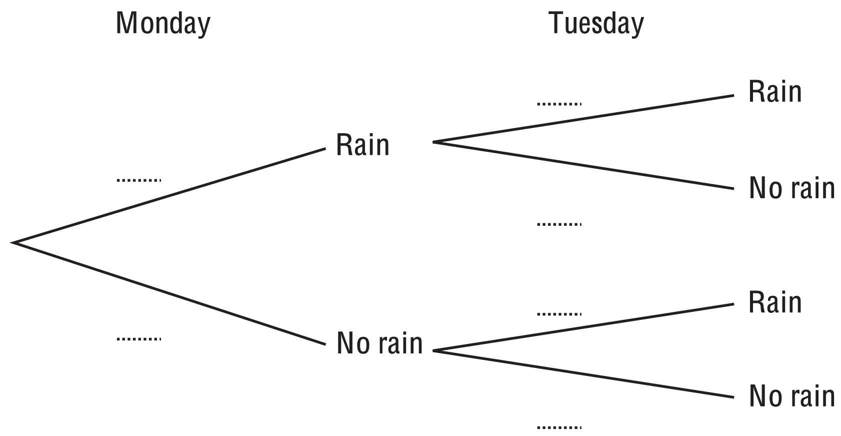
2 In this question give all your answers as fractions.

The probability that it rains on Monday is $\frac{3}{5}$.

If it rains on Monday, the probability that it rains on Tuesday is $\frac{4}{7}$.

If it does not rain on Monday, the probability that it rains on Tuesday is $\frac{5}{7}$.

(a) Complete the tree diagram.



[3]

(b) Find the probability that it rains

(i) on **both** days,

Answer(b)(i) .... [2]

(ii) on Monday but not on Tuesday,

Answer(b)(ii) .... [2]

(iii) on **only one** of the two days.

Answer(b)(iii) .... [2]

(c) If it does **not** rain on Monday and it does **not** rain on Tuesday, the probability that it does **not** rain on Wednesday is $\frac{1}{4}$.

Calculate the probability that it rains on **at least one** of the three days.

Answer(c) .... [3]

7 Katrina puts some plants in her garden.

The probability that a plant will produce a flower is $\frac{7}{10}$.

If there is a flower, it can only be red, yellow or orange.

When there is a flower, the probability it is red is $\frac{2}{3}$ and the probability it is yellow is $\frac{1}{4}$.

(a) Draw a tree diagram to show **all** this information.

Label the diagram and write the probabilities on each branch.

Answer(a)

[5]

(b) A plant is chosen at random.

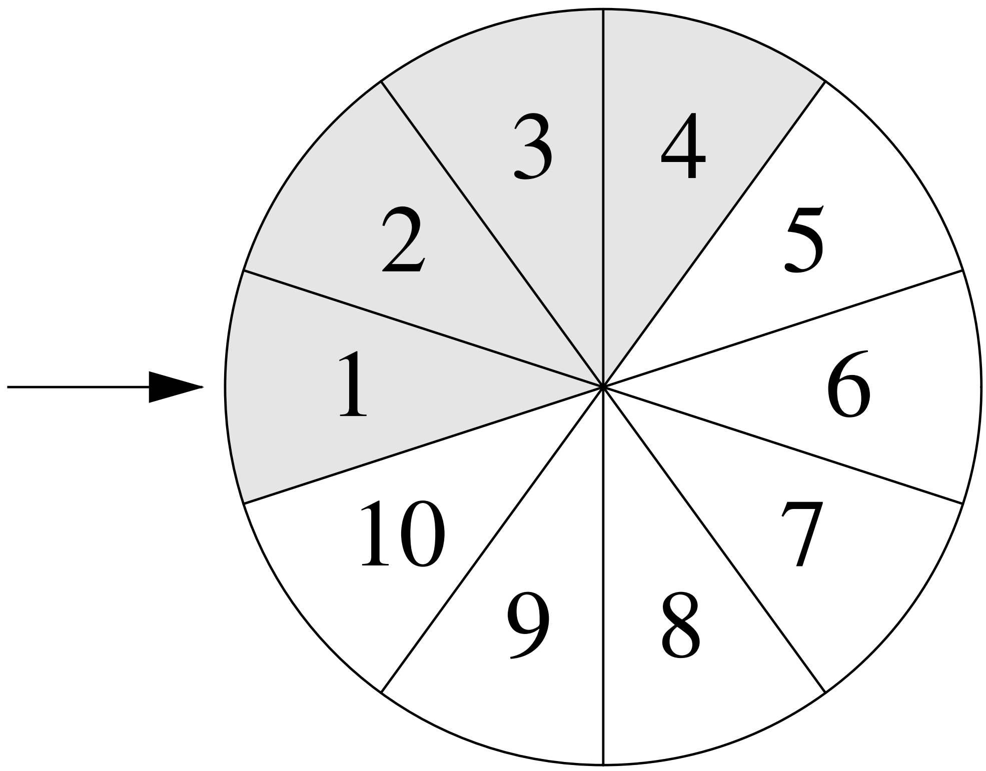
Find the probability that it will **not** produce a yellow flower.

Answer(b) .... [3]

(c) If Katrina puts 120 plants in her garden, how many orange flowers would she expect?

Answer(c) .... [2]

4



A wheel is divided into 10 sectors numbered 1 to 10 as shown in the diagram.
The sectors 1, 2, 3 and 4 are shaded.
The wheel is spun and when it stops the fixed arrow points to one of the sectors.
(Each sector is equally likely.)

(a) The wheel is spun once so that one sector is selected. Find the probability that

(i) the number in the sector is even, [1]

(ii) the sector is shaded, [1]

(iii) the number is even **or** the sector is shaded, [1]

(iv) the number is odd **and** the sector is shaded. [1]

(b) The wheel is spun twice so that each time a sector is selected. Find the probability that

(i) both sectors are shaded, [2]

(ii) one sector is shaded and one is not, [2]

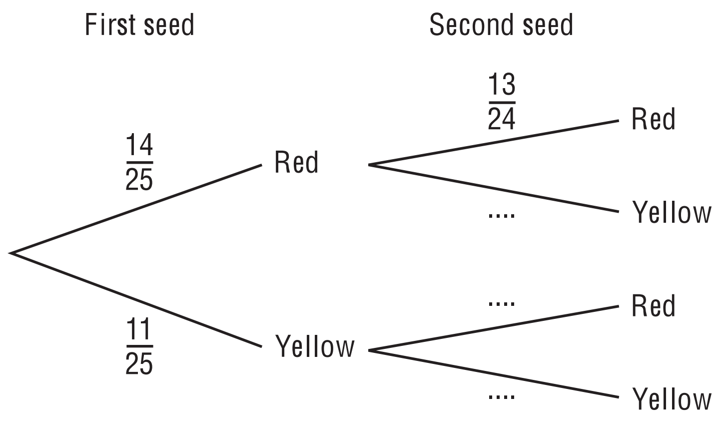
(iii) the sum of the numbers in the two sectors is greater than 20, [2]

(iv) the sum of the numbers in the two sectors is less than 4, [2]

(v) the product of the numbers in the two sectors is a square number. [3]

20 A gardener plants seeds from a packet of 25 seeds.
14 of the seeds will give red flowers and 11 will give yellow flowers.
The gardener chooses two seeds at random.

(a) Write the missing probabilities on the tree diagram below.



[2]

(b) What is the probability that the gardener chooses two seeds which will give

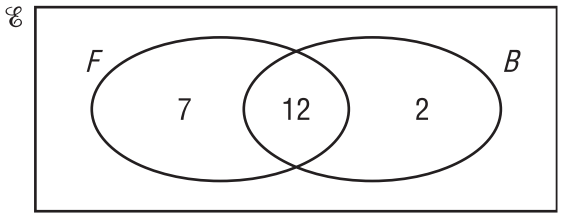
(i) two red flowers,

Answer(b)(i) .... [2]

(ii) two flowers of a different colour?

Answer(b)(ii) .... [2]

4 **(a)** All 24 students in a class are asked whether they like football and whether they like basketball.
Some of the results are shown in the Venn diagram below.



$\mathscr{E}$ = {students in the class}.
F = {students who like football}.
B = {students who like basketball}.

(i) How many students like both sports? [1]

(ii) How many students do not like either sport? [1]

(iii) Write down the value of $n(F \cup B)$. [1]

(iv) Write down the value of $n(F' \cap B)$. [1]

(v) A student from the class is selected at random.
What is the probability that this student likes basketball? [1]

(vi) A student who likes football is selected at random.
What is the probability that this student likes basketball? [1]

(b) Two students are selected at random from a group of 10 boys and 12 girls.
Find the probability that

(i) they are both girls, [2]

(ii) one is a boy and one is a girl. [3]

(i) Write down, as fractions, the values of s, t and u. [3]
(ii) Calculate the probability that it rains on both days. [2]
(iii) Calculate the probability that it will not rain tomorrow. [2]

(b) Each time Christina throws a ball at a target, the probability that she hits the target is $\frac{1}{3}$.

She throws the ball three times.
Find the probability that she hits the target

(i) three times, [2]
(ii) at least once. [2]

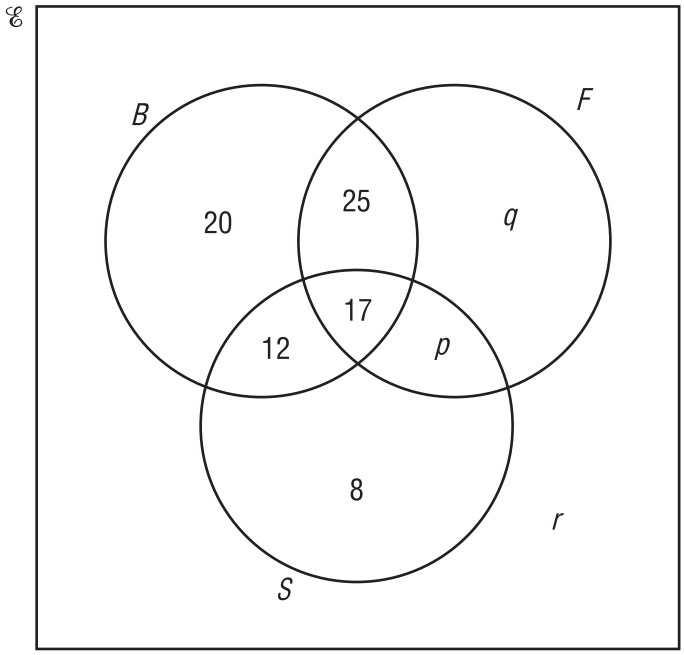
(c) Each time Eduardo throws a ball at the target, the probability that he hits the target is $\frac{1}{4}$.

He throws the ball until he hits the target.
Find the probability that he **first** hits the target with his

(i) 4th throw, [2]
(ii) nth throw. [1]

9 In a survey, 100 students are asked if they like basketball (B), football (F) and swimming (S).

The Venn diagram shows the results.



42 students like swimming.

40 students like exactly one sport.

(a) Find the values of p, q and r. [3]

(b) How many students like

(i) all three sports, [1]

(ii) basketball and swimming but not football? [1]

(c) Find

(i) $n(B')$, [1]

(ii) $n((B \cup F) \cap S')$. [1]

(d) One student is chosen at random from the 100 students.
Find the probability that the student

(i) only likes swimming, [1]

(ii) likes basketball but not swimming. [1]

(e) Two students are chosen at random from those who like basketball.

Find the probability that they each like exactly one other sport. [3]

3

1	1	6	7	11	12

Six cards are numbered 1, 1, 6, 7, 11 and 12.

In this question, give all probabilities as fractions.

(a) One of the six cards is chosen at random.

(i) Which number has a probability of being chosen of $\frac{1}{3}$?

Answer(a)(i) .... [1]

(ii) What is the probability of choosing a card with a number which is smaller than **at least three of the other numbers**?

Answer(a)(ii) .... [1]

(b) Two of the six cards are chosen at random, without replacement.

Find the probability that

(i) they are both numbered 1,

Answer(b)(i) .... [2]

(ii) the total of the two numbers is 18,

Answer(b)(ii) .... [3]

(iii) the first number **is not** a 1 and the second number **is** a 1.

Answer(b)(iii) .... [2]

(c) Cards are chosen, without replacement, until a card numbered 1 is chosen.

Find the probability that this happens before the third card is chosen.

Answer(c) .... [2]

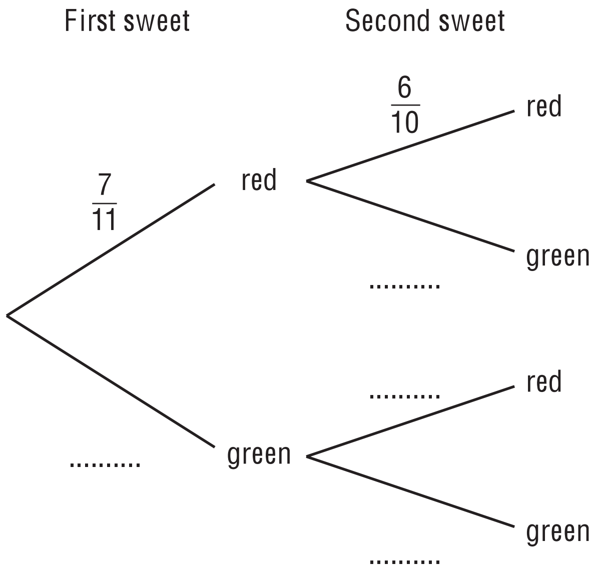
(d) A seventh card is added to the six cards shown in the diagram.
The mean value of the seven numbers on the cards is 6.

Find the number on the seventh card.

Answer(d) .... [2]

9 A bag contains 7 red sweets and 4 green sweets.
Aimee takes out a sweet at random and eats it.
She then takes out a second sweet at random and eats it.

(a) Complete the tree diagram.



[3]

(b) Calculate the probability that Aimee has taken

(i) two red sweets,

Answer(b)(i) .... [2]

(ii) one sweet of each colour.

Answer(b)(ii) .... [3]

(c) Aimee takes a third sweet at random.
Calculate the probability that she has taken

(i) three red sweets,

Answer(c)(i) .... [2]

(ii) at least one red sweet.

Answer(c)(ii) .... [3]