Transformations
2002 - 2011

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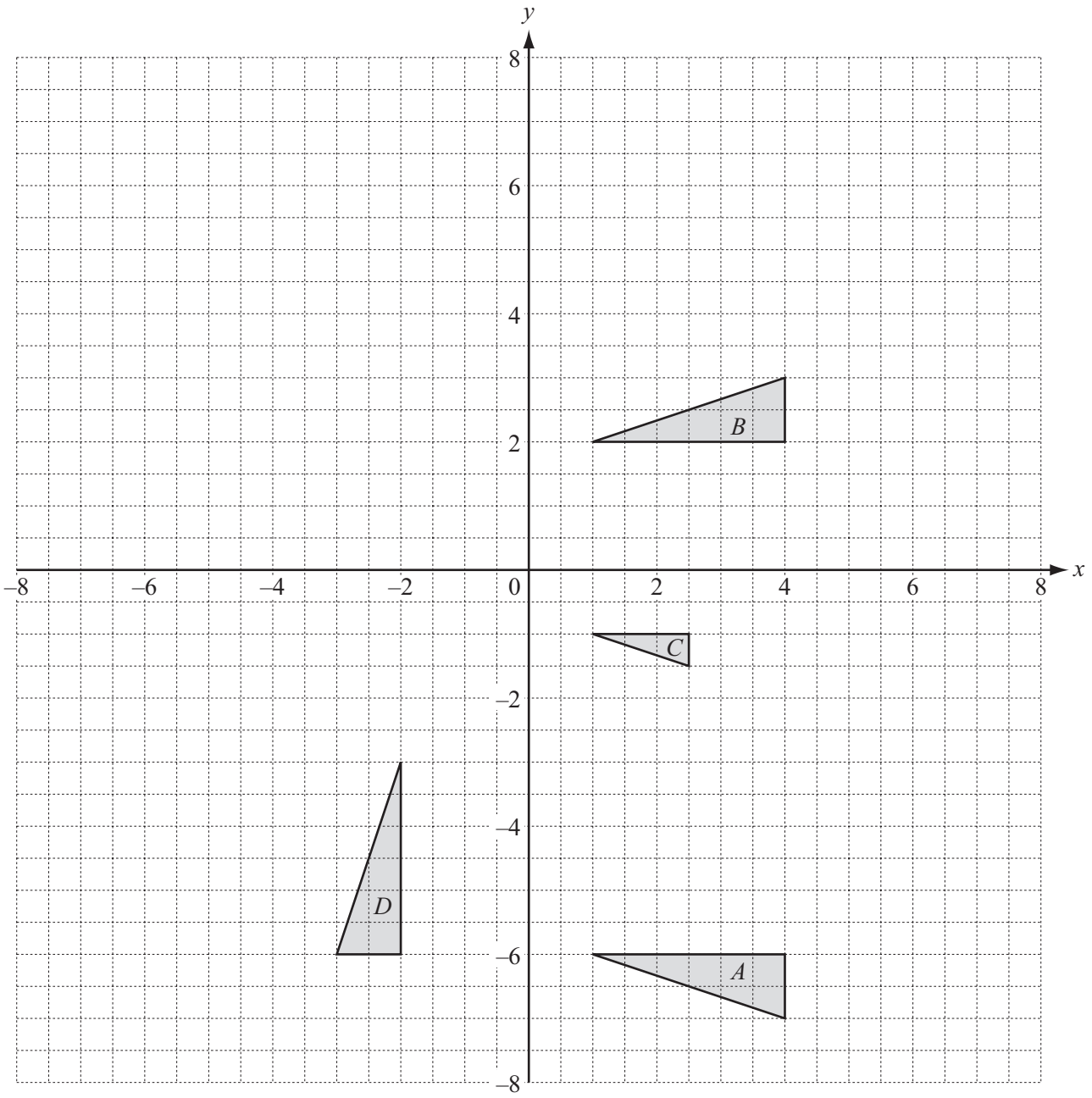


(a) Describe fully the **single** transformation which maps

(i) triangle *A* onto triangle *B*,

Answer(a)(i) ..... [2]

(ii) triangle *A* onto triangle *C*,

Answer(a)(ii) ..... [3]

(iii) triangle *A* onto triangle *D*.

Answer(a)(iii) ..... [3]

(b) Draw the image of

(i) triangle B after a translation of $\begin{pmatrix} -5 \\ 2 \end{pmatrix}$, [2]

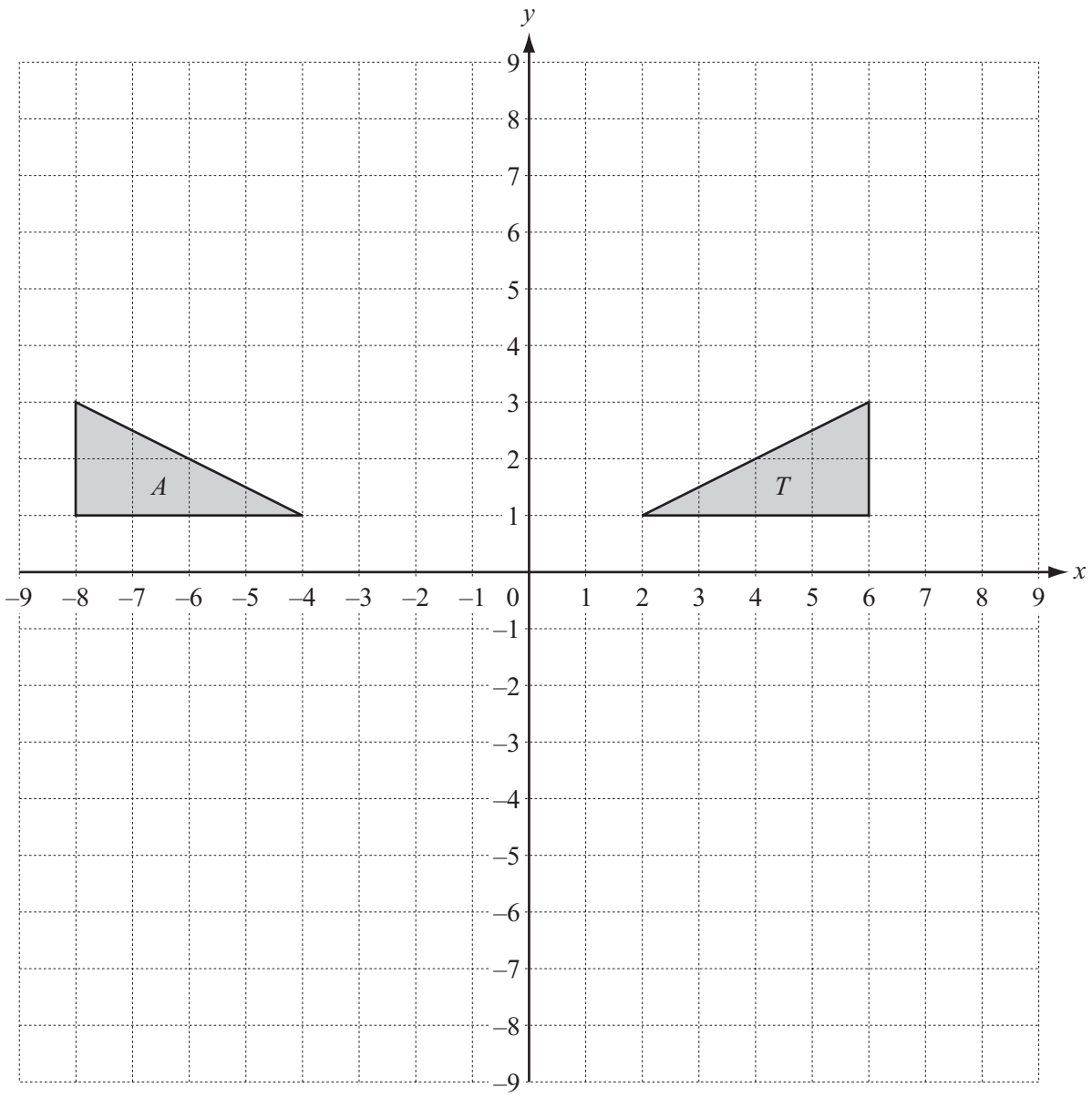
(ii) triangle B after a transformation by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$. [3]

(c) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

Answer(c) .....

..... [3]

3



Triangles T and A are drawn on the grid above.

(a) Describe fully the **single** transformation that maps triangle T onto triangle A.

Answer(a) ..... [2]

(b) (i) Draw the image of triangle T after a rotation of 90° anticlockwise about the point (0,0).

Label the image B. [2]

(ii) Draw the image of triangle T after a reflection in the line $x + y = 0$.

Label the image C. [2]

(iii) Draw the image of triangle T after an enlargement with centre (4, 5) and scale factor 1.5.

Label the image D. [2]

(c) **(i)** Triangle T has its vertices at co-ordinates (2, 1), (6, 1) and (6, 3).

Transform triangle T by the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

Draw this image on the grid and label it E.

[3]

(ii) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$.

Answer(c)(ii) ..... [3]

(d) Write down the matrix that transforms triangle B onto triangle T.

Answer(d) $\begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix}$ [2]

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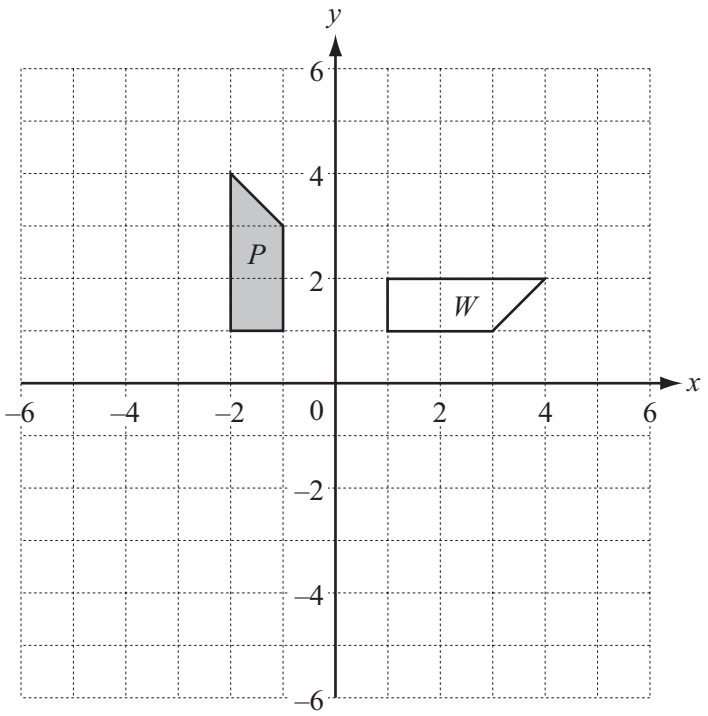


(a) Draw the reflection of shape P in the line $y = x$. [2]

(b) Draw the translation of shape P by the vector $\begin{pmatrix} -2 \\ 1 \end{pmatrix}$. [2]

(c) **(i)** Describe fully the **single** transformation that maps shape P onto shape W.

Answer(c)(i) ..... [3]

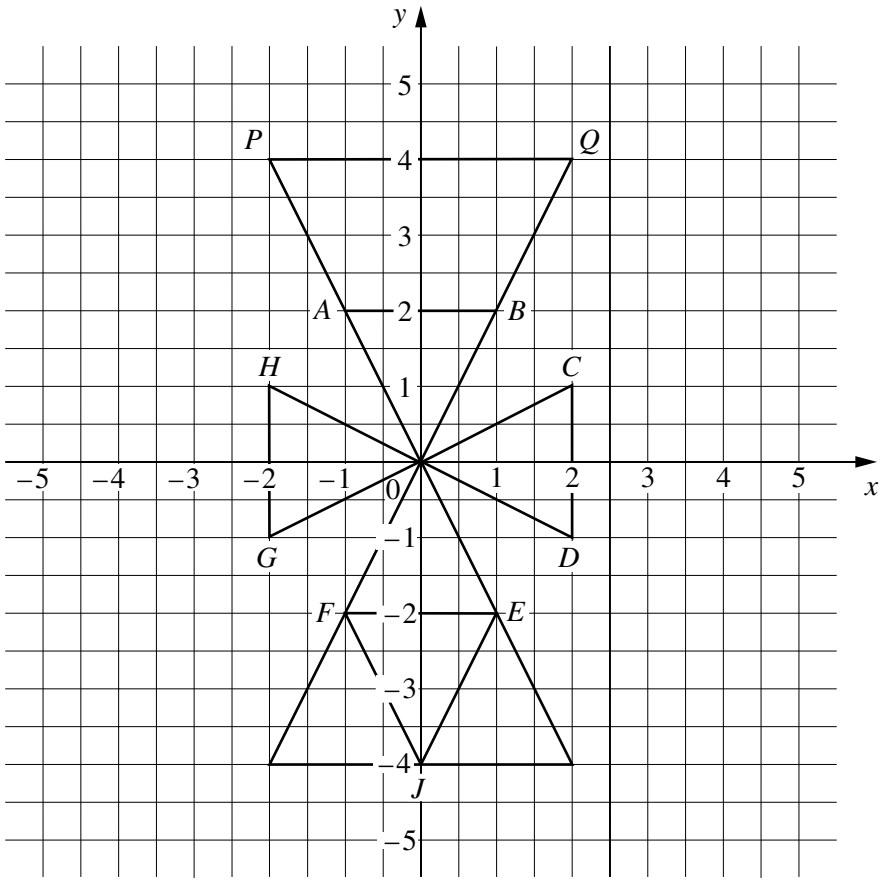
(ii) Find the 2 by 2 matrix which represents this transformation.

Answer(c)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(d) Describe fully the **single** transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$.

Answer(d) ..... [3]

7



(a) Describe fully a single transformation which maps both

(i) *A* onto *C* **and** *B* onto *D*, [2]

(ii) *A* onto *D* **and** *B* onto *C*, [2]

(iii) *A* onto *P* **and** *B* onto *Q*. [3]

(b) Describe fully a single transformation which maps triangle *0AB* onto triangle *JFE*. [2]

(c) The matrix **M** is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

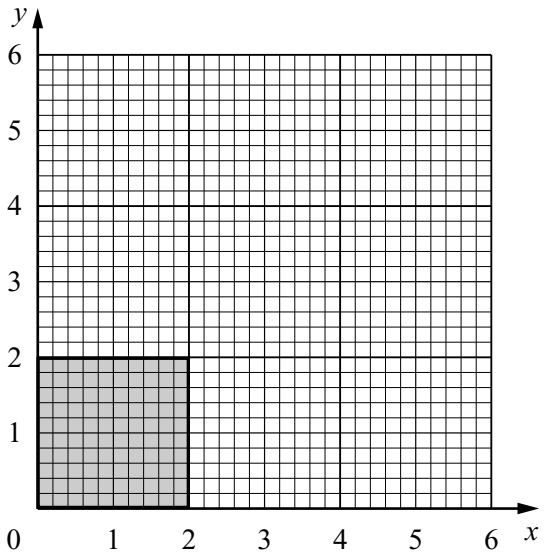
(i) Describe the transformation which **M** represents. [2]

(ii) Write down the co-ordinates of *P* after transformation by matrix **M**. [2]

(d) **(i)** Write down the matrix **R** which represents a rotation by 90° anticlockwise about *0*. [2]

(ii) Write down the letter representing the new position of *F* after the transformation **RM**(*F*). [2]

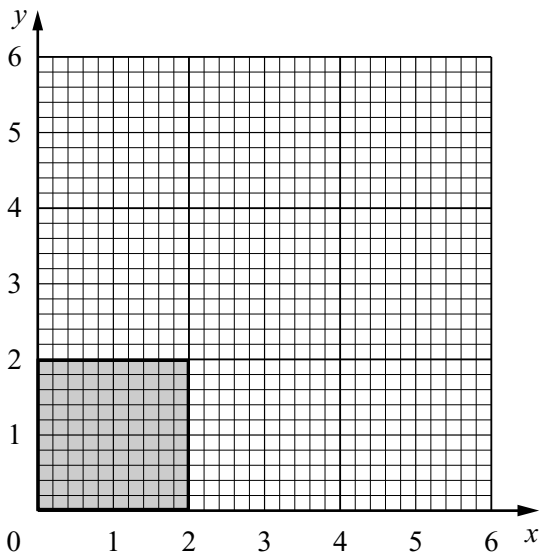
17 (a)



Draw the shear of the shaded square with the x-axis invariant and the point $(0, 2)$ mapping onto the point $(3, 2)$.

[2]

(b)



(i) Draw the one-way stretch of the shaded square with the x-axis invariant and the point $(0, 2)$ mapping onto the point $(0, 6)$.

[2]

(ii) Write down the matrix of this stretch.

Answer (b)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

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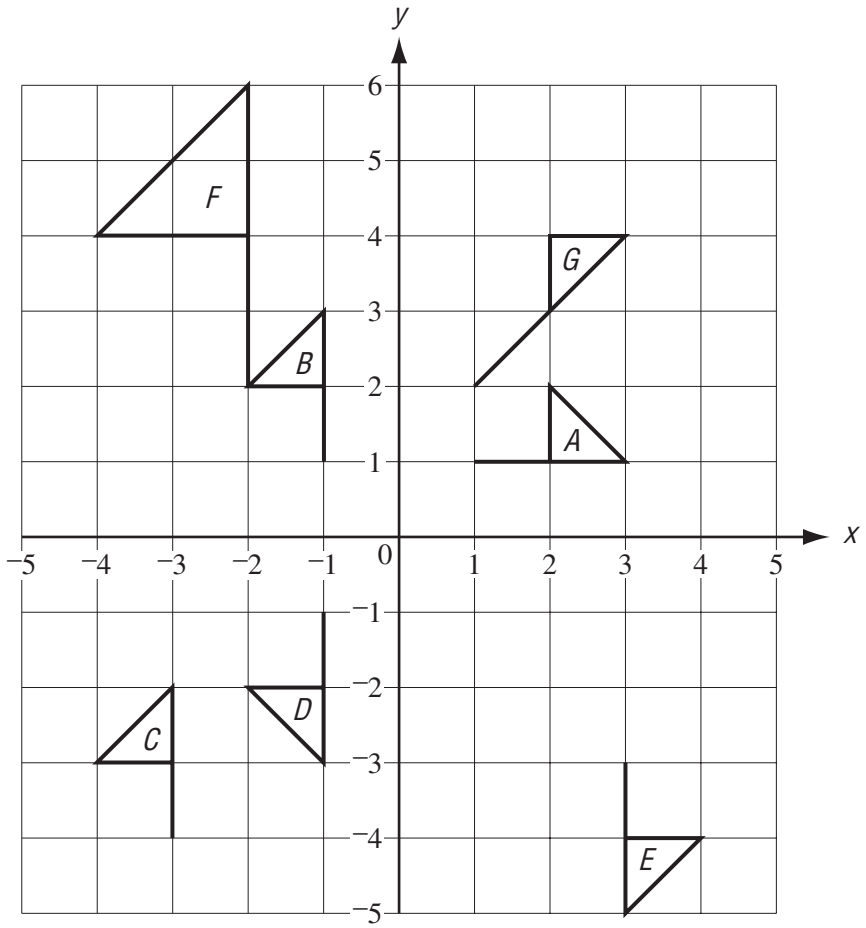

(a) Describe fully the **single** transformation which maps

(i) shape *A* onto shape *B*, [2]

(ii) shape *B* onto shape *C*, [2]

(iii) shape *A* onto shape D, [2]

(iv) shape *B* onto shape *E*, [2]

(v) shape *B* onto shape *F*, [2]

(vi) shape *A* onto shape *G*. [2]

(b) A transformation is represented by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Which shape above is the image of shape *A* after this transformation? [2]

(c) Find the 2 by 2 matrix representing the transformation which maps

(i) shape *B* onto shape *D*, [2]

(ii) shape *A* onto shape *G*. [2]

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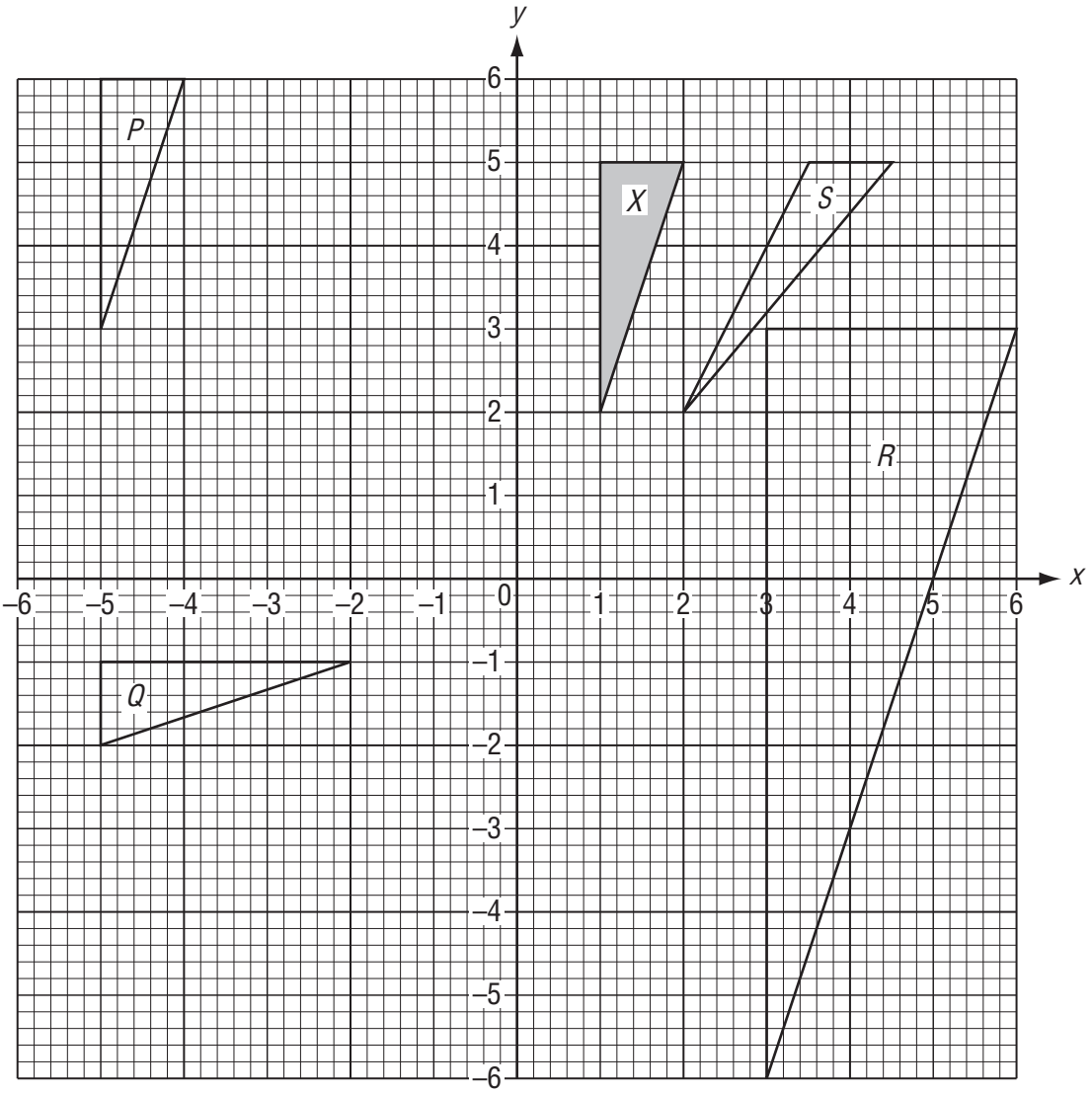


(a) Describe fully the single transformation which maps

(i) triangle X onto triangle P, [2]

(ii) triangle X onto triangle Q, [2]

(iii) triangle X onto triangle R, [3]

(iv) triangle X onto triangle S. [3]

(b) Find the 2 by 2 matrix which represents the transformation that maps

(i) triangle X onto triangle Q, [2]

(ii) triangle X onto triangle S. [2]

7 Transformation T is translation by the vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

Transformation M is reflection in the line $y = x$.

(a) The point A has co-ordinates (2, 1).

Find the co-ordinates of

(i) T(A), [1]

(ii) MT(A). [2]

(b) Find the 2 by 2 matrix **M**, which represents the transformation M. [2]

(c) Show that, for any value of k, the point Q $(k - 2, k - 3)$ maps onto a point on the line $y = x$ following the transformation TM(Q). [3]

(d) Find $\mathbf{M}^{-1}$, the inverse of the matrix **M**. [2]

(e) **N** is the matrix such that $\mathbf{N} + \begin{pmatrix} 0 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ 0 & 0 \end{pmatrix}$.

(i) Write down the matrix **N**. [2]

(ii) Describe completely the **single** transformation represented by **N**. [3]

2 **Answer the whole of this question on a sheet of graph paper.**

(a) Draw and label x and y axes from -6 to 6, using a scale of 1 cm to 1 unit. [1]

(b) Draw triangle ABC with A (2,1), B (3,3) and C (5,1). [1]

(c) Draw the reflection of triangle ABC in the line $y = x$. Label this $A_1B_1C_1$. [2]

(d) Rotate **triangle $A_1B_1C_1$** about (0,0) through 90° anti-clockwise. Label this $A_2B_2C_2$. [2]

(e) Describe fully the single transformation which maps triangle ABC onto triangle $A_2B_2C_2$. [2]

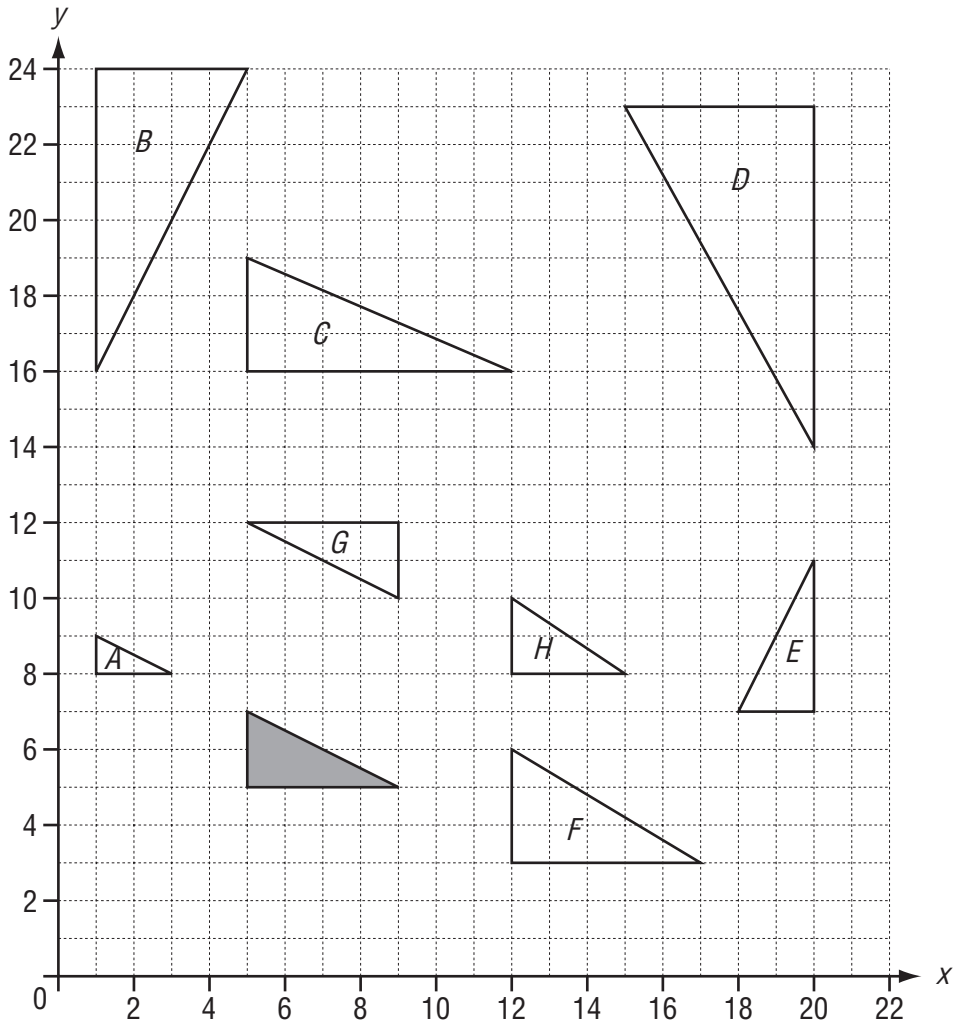
(f) A transformation is represented by the matrix $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$.

(i) Draw the image of triangle ABC under this transformation. Label this $A_3B_3C_3$. [3]

(ii) Describe fully the single transformation represented by the matrix $\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$. [2]

(iii) Find the matrix which represents the transformation that maps triangle $A_3B_3C_3$ onto triangle ABC. [2]

18



Write down the letters of all the triangles which are

(a) congruent to the shaded triangle,

Answer(a) ..... [2]

(b) similar, but not congruent, to the shaded triangle.

Answer(b) ..... [2]

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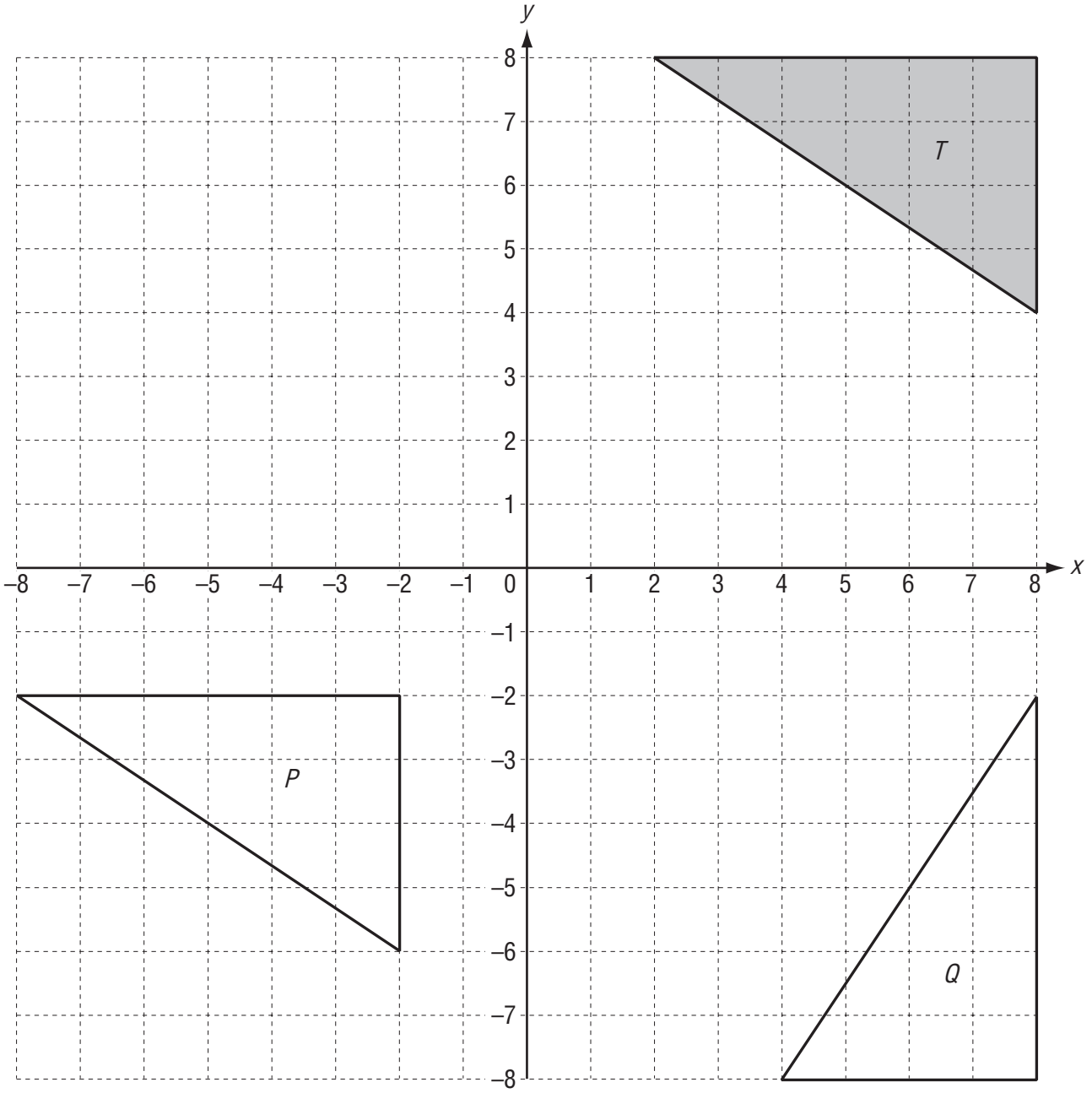

(a) On the grid, draw the enlargement of the triangle T, centre (0, 0), scale factor $\frac{1}{2}$. [2]

(b) The matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation.

(i) Calculate the matrix product $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 8 & 8 & 2 \\ 4 & 8 & 8 \end{pmatrix}$.

Answer(b)(i) [2]

(ii) On the grid, draw the image of the triangle *T* under this transformation. [2]

(iii) Describe fully this **single** transformation.

Answer(b)(iii) ..... [2]

(c) Describe fully the **single** transformation which maps

(i) triangle *T* onto triangle *P*,

Answer(c)(i) ..... [2]

(ii) triangle *T* onto triangle *Q*.

Answer(c)(ii) ..... [3]

(d) Find the 2 by 2 matrix which represents the transformation in **part (c)(ii)**.

Answer(d) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

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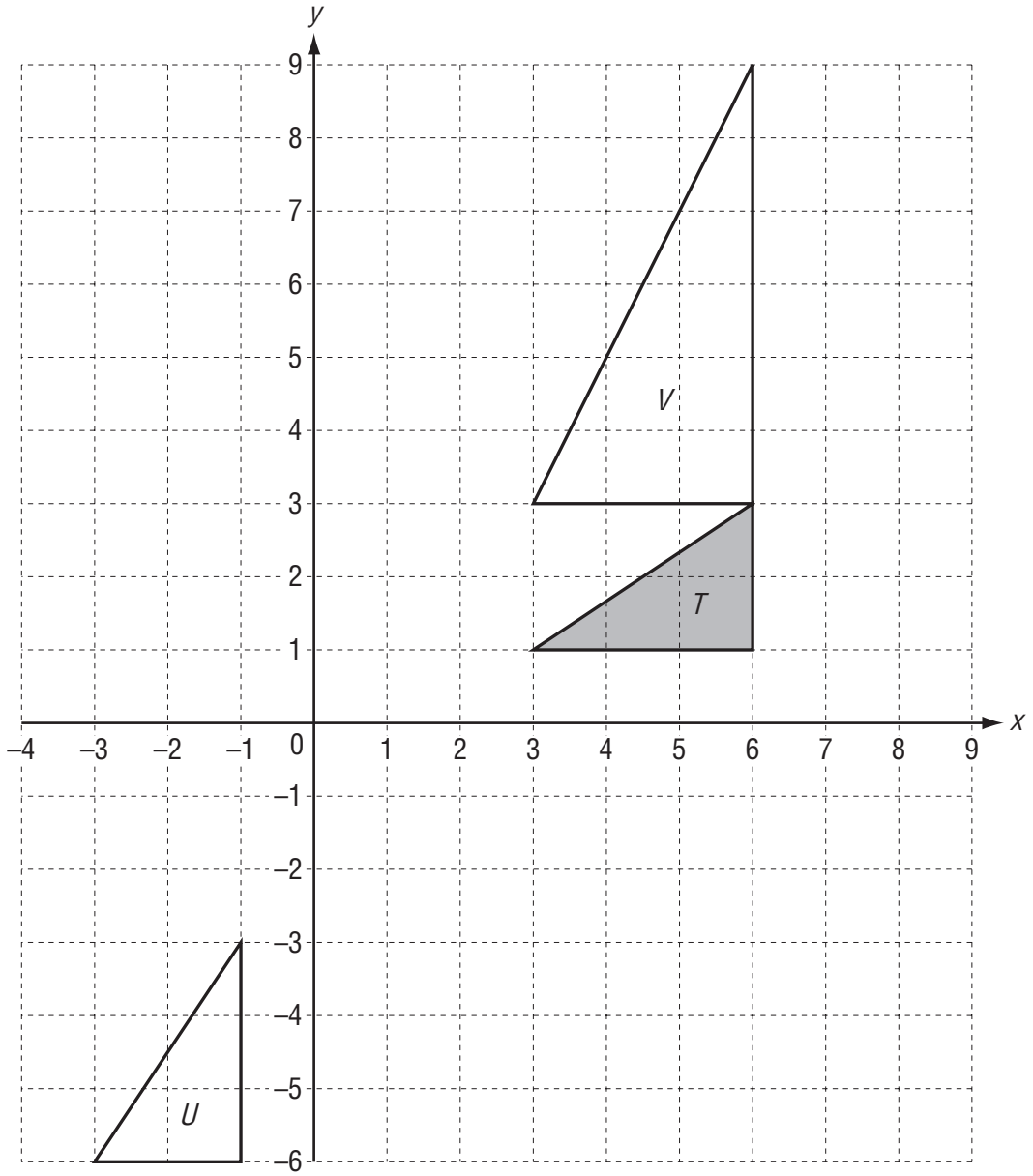

(a) On the grid, draw

(i) the translation of triangle T by the vector $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$, [2]

(ii) the rotation of triangle T about (0, 0), through 90° clockwise. [2]

(b) Describe fully the **single** transformation that maps

(i) triangle T onto triangle U,

Answer(b)(i) ..... [2]

(ii) triangle T onto triangle V.

Answer(b)(ii) ..... [3]

(c) Find the 2 by 2 matrix which represents the transformation that maps

(i) triangle T onto triangle U,

Answer(c)(i) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) triangle T onto triangle V,

Answer(c)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(iii) triangle V onto triangle T.

Answer(c)(iii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [1]

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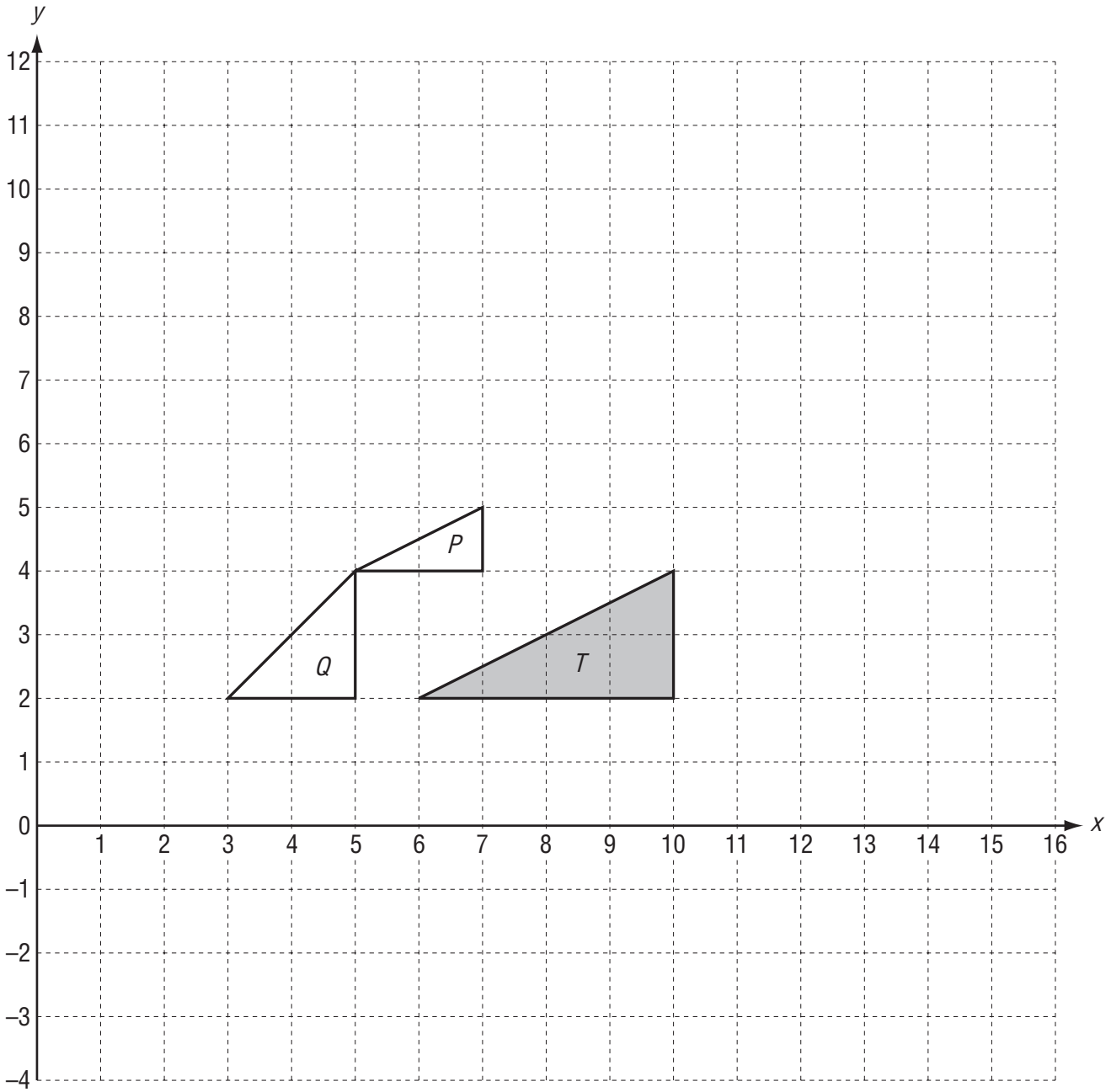


(a) Draw the reflection of triangle T in the line $y = 6$.

Label the image A. [2]

(b) Draw the translation of triangle T by the vector $\begin{pmatrix} -4 \\ 6 \end{pmatrix}$.

Label the image B. [2]

(c) Describe fully the **single** transformation which maps triangle *B* onto triangle *T*.

Answer(c) ..... [2]

(d) (i) Describe fully the **single** transformation which maps triangle *T* onto triangle *P*.

Answer(d)(i) ..... [3]

(ii) Complete the following statement.

Area of triangle *P* = ... × Area of triangle *T* [1]

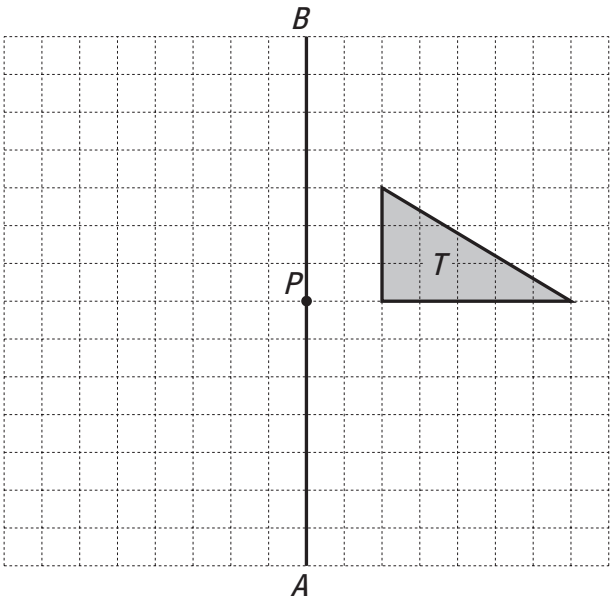
(e) (i) Describe fully the **single** transformation which maps triangle *T* onto triangle *Q*.

Answer(e)(i) ..... [3]

(ii) Find the 2 by 2 matrix which represents the transformation mapping triangle *T* onto triangle *Q*.

Answer(e)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

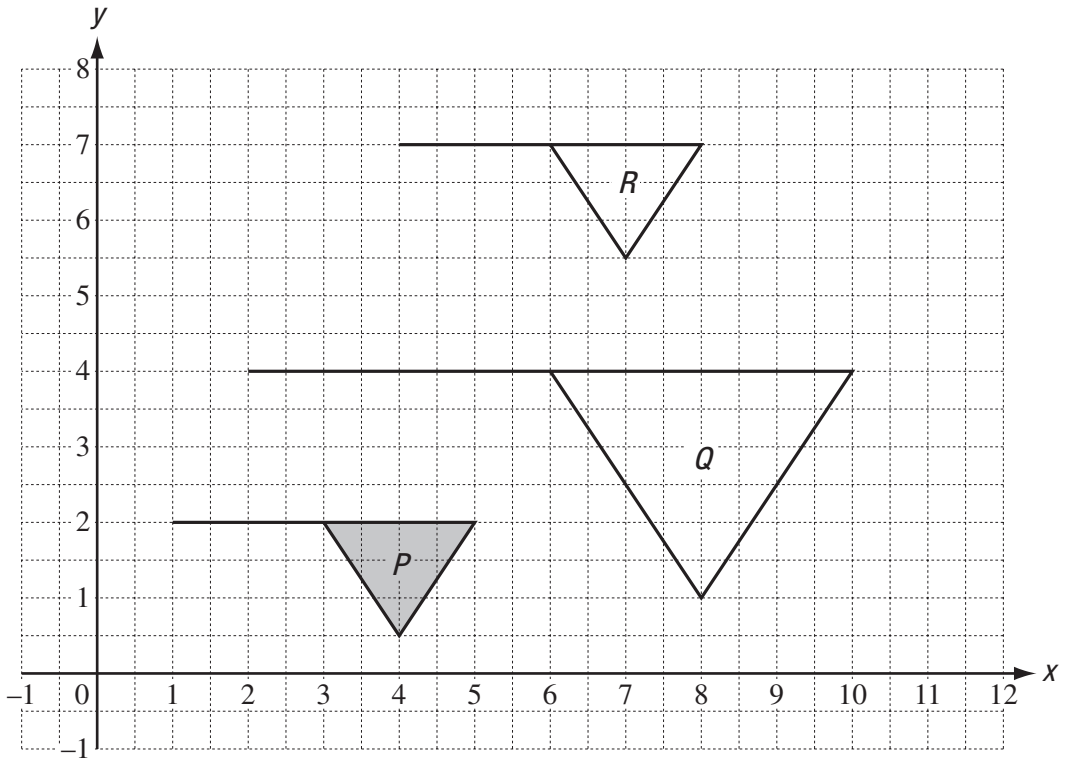
7 **(a)**



(i) Reflect triangle T in the line AB.
Label your image X. [1]

(ii) Rotate triangle T through 90° clockwise about the point P.
Label your image Y. [2]

(b)



Describe the **single** transformation which maps

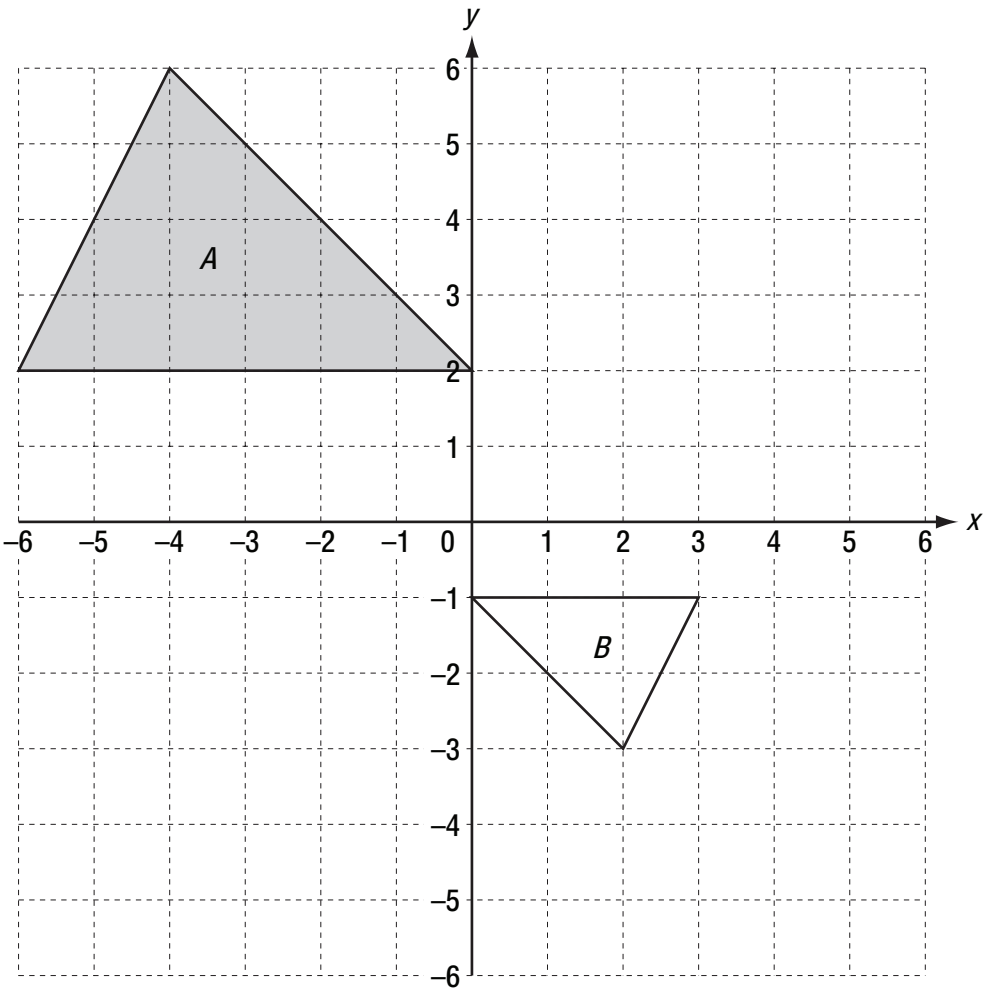
(i) flag P onto flag Q,

Answer(b)(i) ..... [3]

(ii) flag P onto flag R.

Answer(b)(ii) ..... [2]

2



The diagram shows two triangles drawn on a 1 cm square grid.

(a) **(i)** Describe fully the **single** transformation which maps triangle *A* onto triangle *B*.

Answer(a)(i) ..... [3]

(ii) Calculate the area of triangle *A*.

Answer(a)(ii) ..... cm^2 [2]

(iii) Find the perimeter of triangle *A*.

Answer(a)(iii) ..... cm [1]

(b) Reflect triangle *A* in the *x*-axis.
Label the image *P*. [1]

(c) Rotate triangle *A* through 90° clockwise about (0, 0).
Label the image *Q*. [2]

(d) Describe fully the **single** transformation which maps triangle *P* onto triangle *Q*.

Answer(d) ..... [2]

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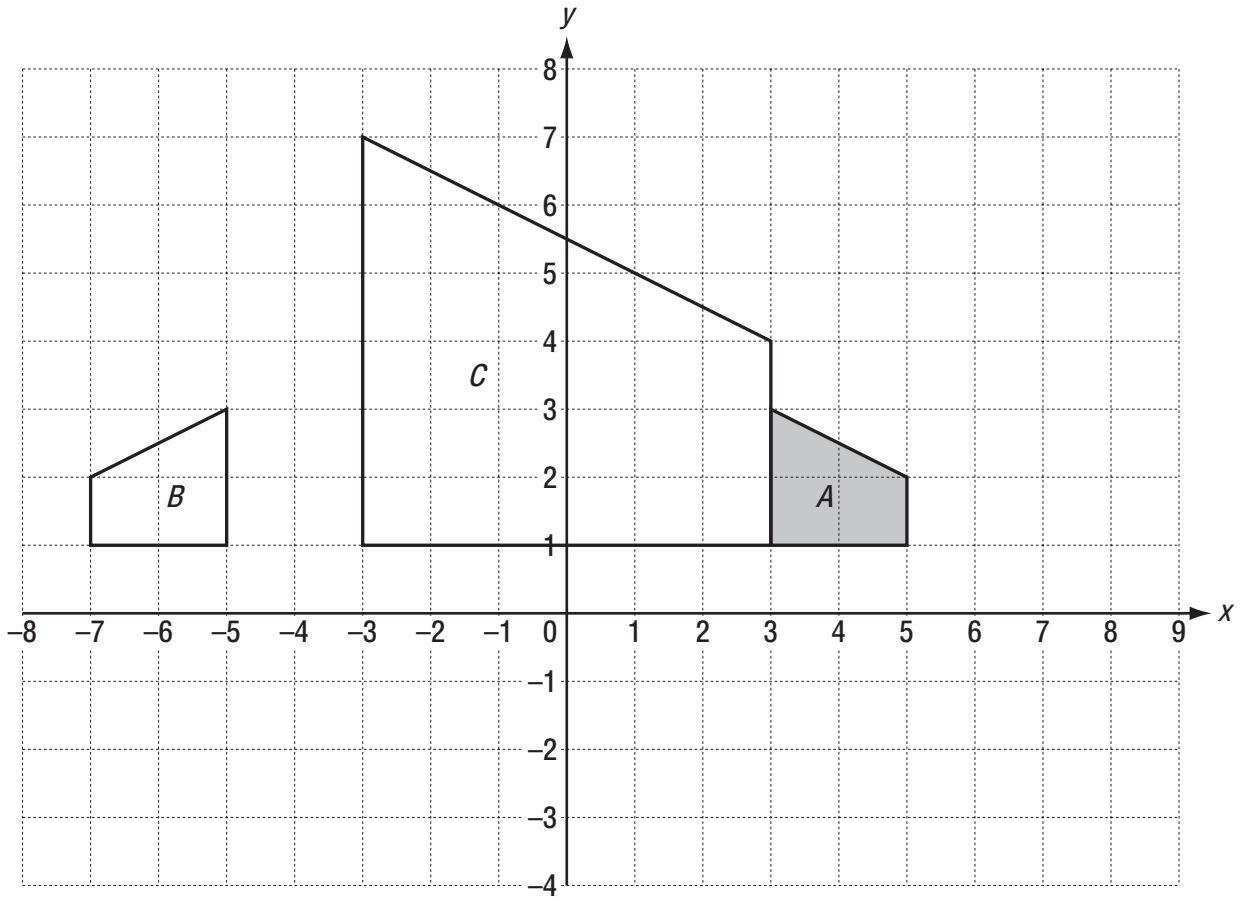


(a) On the grid, draw the images of the following transformations of **shape *A***.

(i) Reflection in the *x*-axis [1]

(ii) Translation by the vector $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ [2]

(iii) Rotation, centre (0, 0), through 180° [2]

(b) Describe fully the **single** transformation that maps

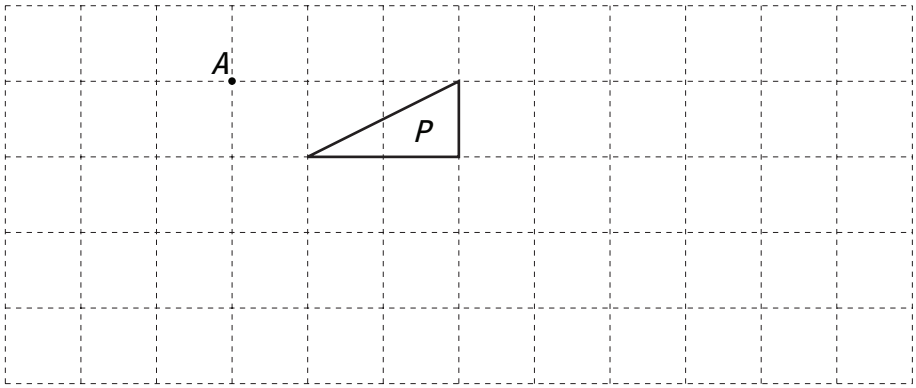
(i) shape *A* onto shape *B*,

Answer(b)(i) ..... [2]

(ii) shape *A* onto shape *C*.

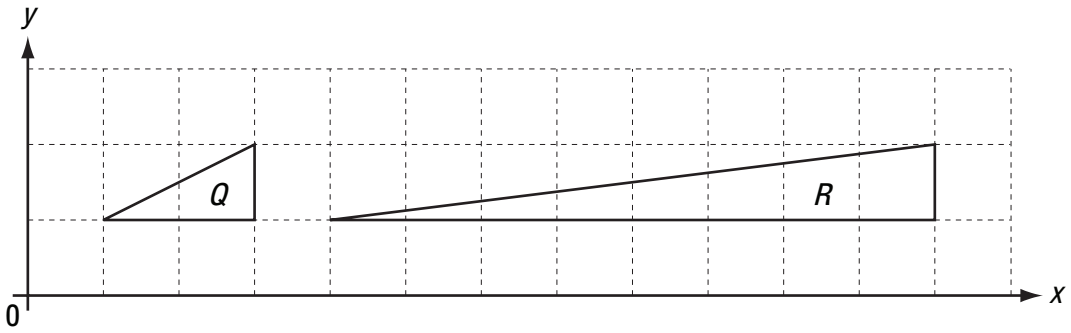
Answer(b)(ii) ..... [3]

8 **(a)**



Draw the enlargement of triangle P with centre A and scale factor 2. [2]

(b)



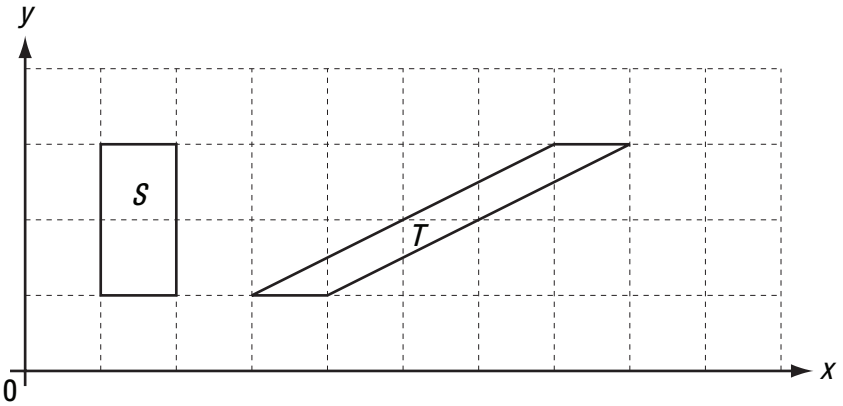
(i) Describe fully the **single** transformation which maps shape Q onto shape R.

Answer(b)(i) ..... [3]

(ii) Find the matrix which represents this transformation.

Answer(b)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

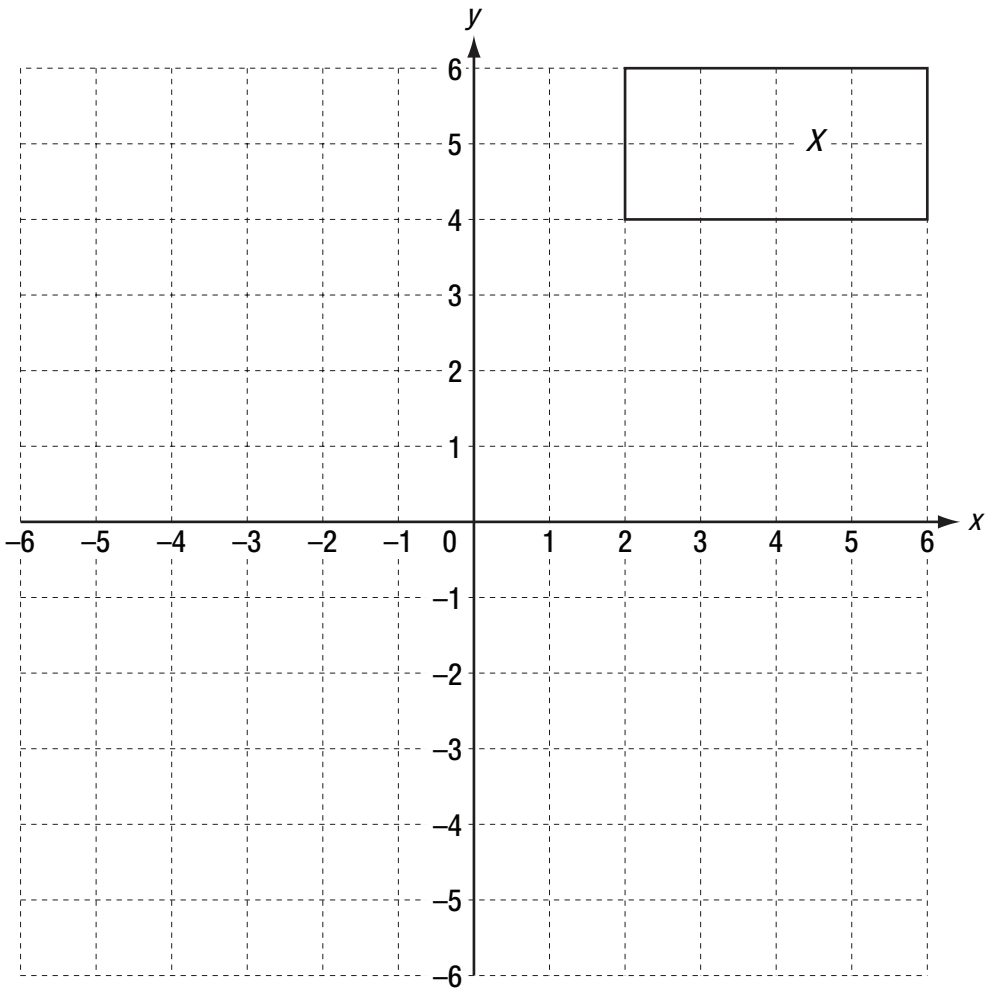
(c)



Describe fully the **single** transformation which maps shape S onto shape T.

Answer(c) ..... [3]

2



(a) (i) Draw the reflection of shape *X* in the *x*-axis. Label the image *Y*. [2]

(ii) Draw the rotation of **shape *Y***, 90° clockwise about (0, 0). Label the image *Z*. [2]

(iii) Describe fully the **single** transformation that maps shape *Z* onto shape *X*.

Answer(a)(iii) ..... [2]

(b) (i) Draw the enlargement of shape *X*, centre (0, 0), scale factor $\frac{1}{2}$. [2]

(ii) Find the matrix which represents an enlargement, centre (0, 0), scale factor $\frac{1}{2}$.

Answer(b)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(c) (i) Draw the shear of **shape *X*** with the *x*-axis invariant and shear factor –1. [2]

(ii) Find the matrix which represents a shear with the *x*-axis invariant and shear factor –1.

Answer(c)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

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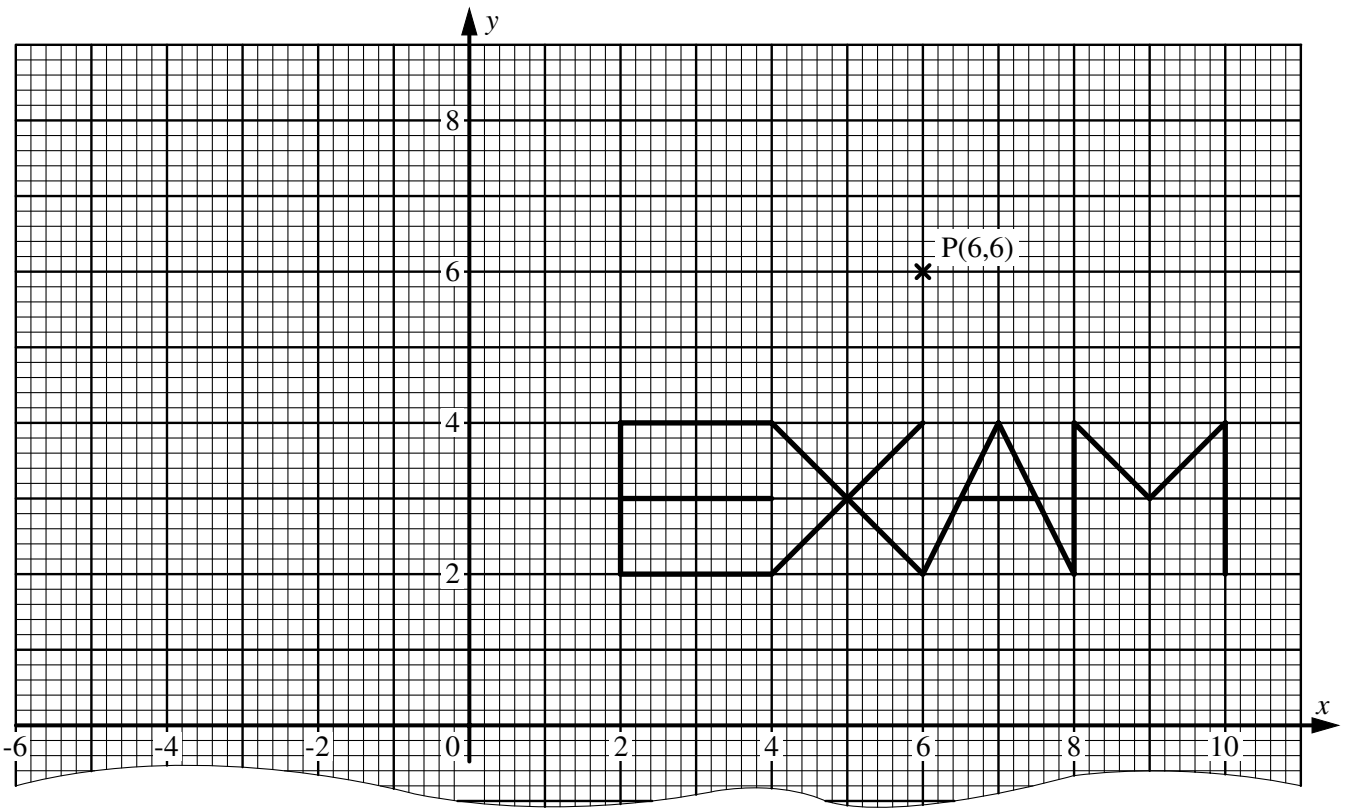


Answer the whole of this question on a sheet of graph paper.

(a) Using a scale of 1 cm to represent 1 unit on each axis, draw an x-axis for $-6 \leqslant x \leqslant 10$ and a y-axis for $-8 \leqslant y \leqslant 8$.
Copy the word EXAM onto your grid so that it is **exactly** as it is in the diagram above.
Mark the point P (6,6). [2]

(b) Draw accurately the following transformations.

(i) Reflect the letter **E** in the line $x = 0$. [2]

(ii) Enlarge the letter **X** by scale factor 3 about centre P (6,6). [2]

(iii) Rotate the letter **A** 90° anticlockwise about the origin. [2]

(iv) Stretch the letter **M** vertically with scale factor 2 and x-axis invariant. [2]

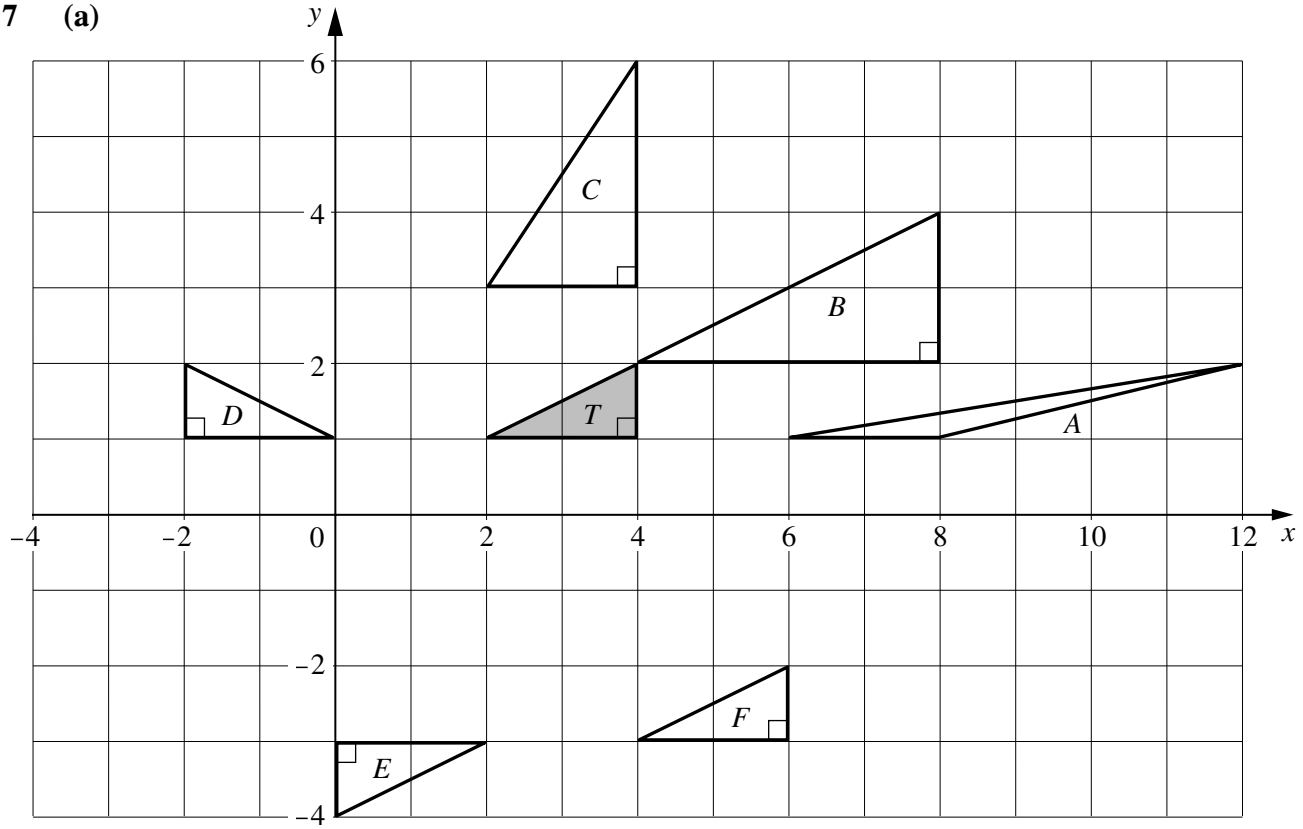
(c) **(i)** Mark and label the point Q so that $\overrightarrow{PQ} = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$. [1]

(ii) Calculate $|\overrightarrow{PQ}|$ correct to two decimal places. [2]

(iii) Mark and label the point S so that $\overrightarrow{PS} \begin{pmatrix} -4 \\ -1 \end{pmatrix}$. [1]

(iv) Mark and label the point R so that $PQRS$ is a parallelogram. [1]

7 **(a)**



Use one of the letters A, B, C, D, E or F to answer the following questions.

(i) Which triangle is T mapped onto by a **translation**? Write down the translation vector. [2]

(ii) Which triangle is T mapped onto by a **reflection**? Write down the equation of the mirror line. [2]

(iii) Which triangle is T mapped onto by a **rotation**? Write down the coordinates of the centre of rotation. [2]

(iv) Which triangle is T mapped onto by a **stretch** with the x-axis invariant? Write down the scale factor of the stretch. [2]

(v) $\mathbf{M} = \begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix}$. Which triangle is T mapped onto by $\mathbf{M}$?

Write down the name of this transformation. [2]

(b) $\mathbf{P} = \begin{pmatrix} 1 & 3 \\ 5 & 7 \end{pmatrix}$, $\mathbf{Q} = (-1 \quad -2)$, $\mathbf{R} = (1 \quad 2 \quad 3)$, $\mathbf{S} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$.

Only some of the following matrix operations are possible with matrices **P**, **Q**, **R** and **S** above.

PQ, **QP**, **P** + **Q**, **PR**, **RS**

Write down and calculate each matrix operation that is possible. [6]

7 **Answer the whole of this question on a sheet of graph paper.**

(a) Draw x and y axes from 0 to 12 using a scale of 1 cm to 1 unit on each axis. [1]

(b) Draw and label triangle T with vertices (8, 6), (6, 10) and (10, 12). [1]

(c) Triangle T is reflected in the line $y = x$.

(i) Draw the image of triangle T. Label this image P. [2]

(ii) Write down the matrix which represents this reflection. [2]

(d) A transformation is represented by the matrix $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

(i) Draw the image of triangle T under this transformation. Label this image Q. [2]

(ii) Describe fully this single transformation. [3]

(e) Triangle T is stretched with the y-axis invariant and a stretch factor of $\frac{1}{2}$.

Draw the image of triangle T. Label this image R. [2]

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8 $f(x) = 2x - 1$, $g(x) = \frac{3}{x} + 1$, $h(x) = 2^x$.

(a) Find the value of fg(6). [1]

(b) Write, as a **single fraction**, gf(x) in terms of x. [3]

(c) Find $g^{-1}(x)$. [3]

(d) Find hh(3). [2]

(e) Find x when $h(x) = g\left(-\frac{24}{7}\right)$ [2]

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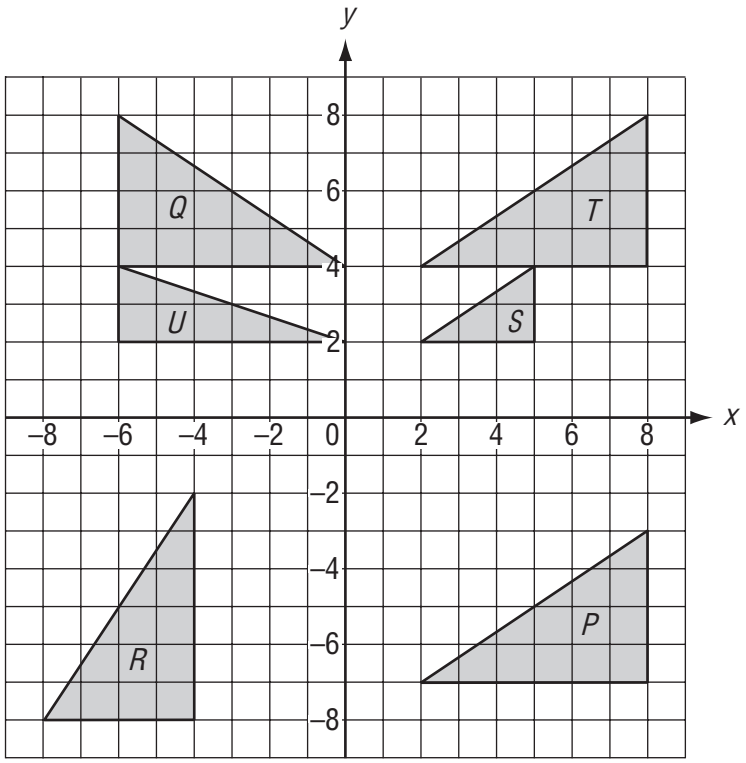


The diagram shows triangles P, Q, R, S, T and U.

(a) Describe fully the **single** transformation which maps triangle

(i) T onto P, [2]

(ii) Q onto T, [2]

(iii) T onto R, [2]

(iv) T onto S, [3]

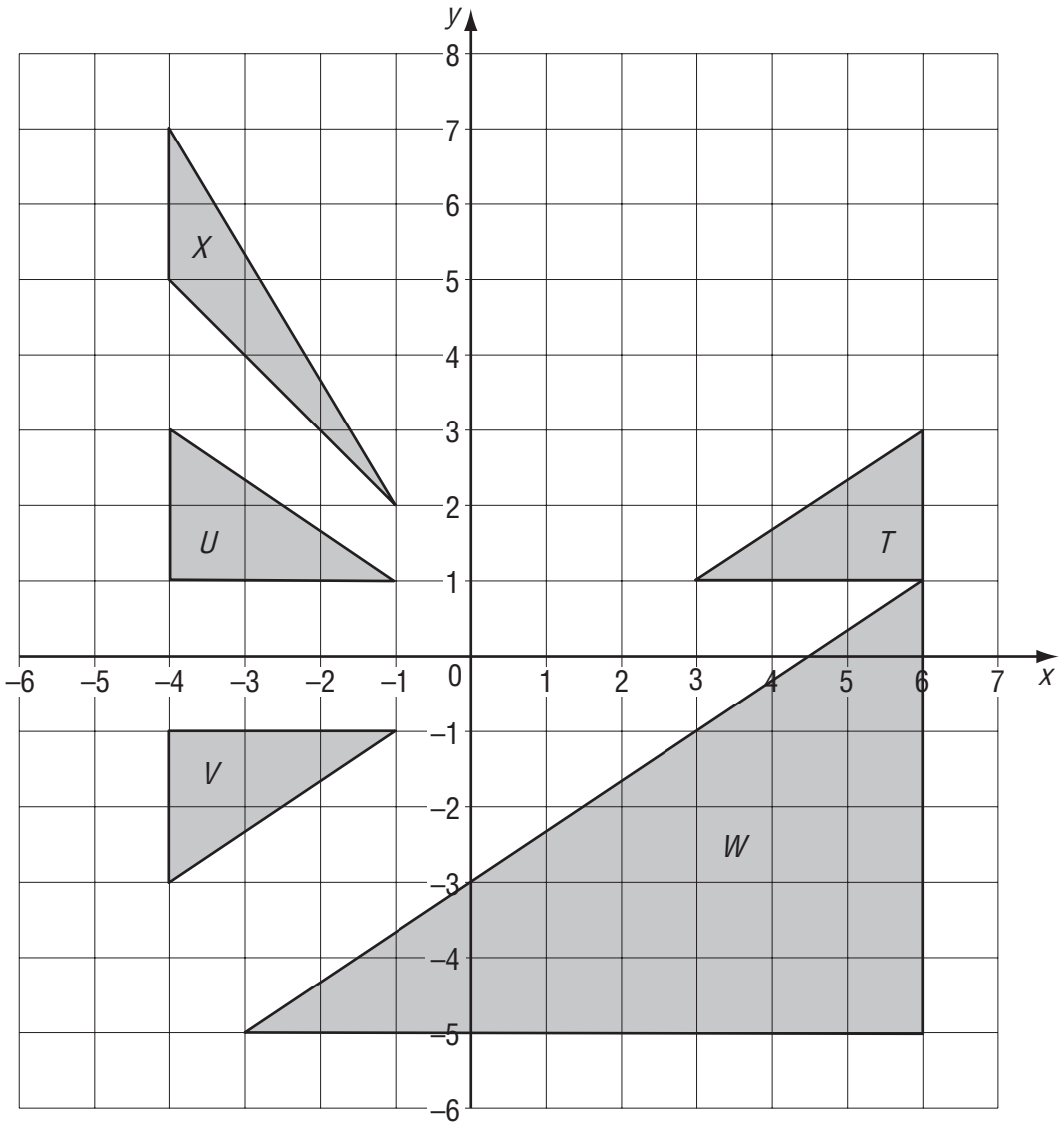
(v) U onto Q. [3]

(b) Find the 2 by 2 matrix representing the transformation which maps triangle

(i) T onto R, [2]

(ii) U onto Q. [2]

2



(a) Describe fully the **single** transformation which maps

(i) triangle *T* onto triangle *U*,

Answer(a)(i) ..... [2]

(ii) triangle *T* onto triangle *V*,

Answer(a)(ii) ..... [3]

(iii) triangle T onto triangle W,

Answer(a)(iii) ..... [3]

(iv) triangle U onto triangle X.

Answer(a)(iv) ..... [3]

(b) Find the matrix representing the transformation which maps

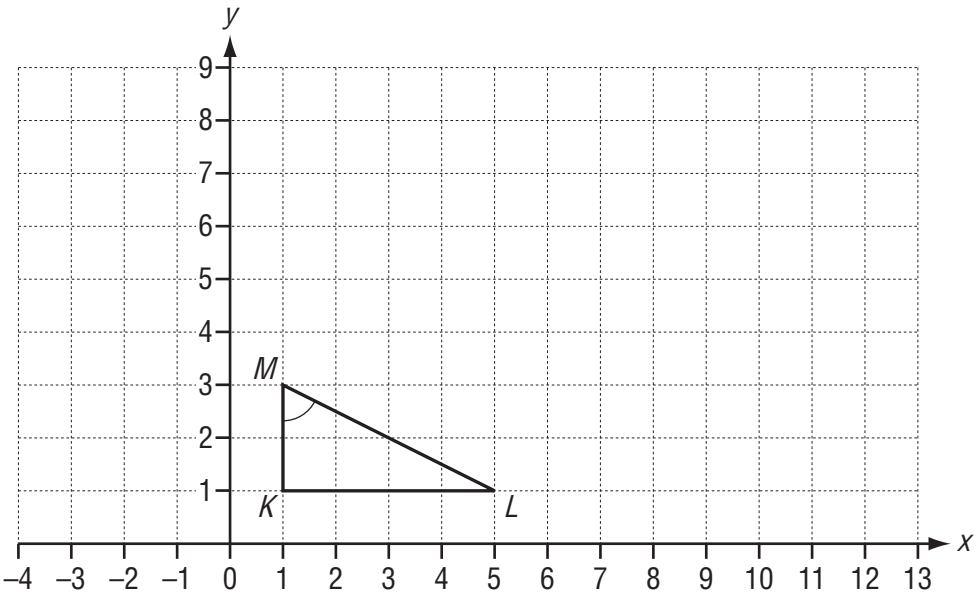
(i) triangle U onto triangle V,

Answer(b)(i) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

(ii) triangle U onto triangle X.

Answer(b)(ii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

20



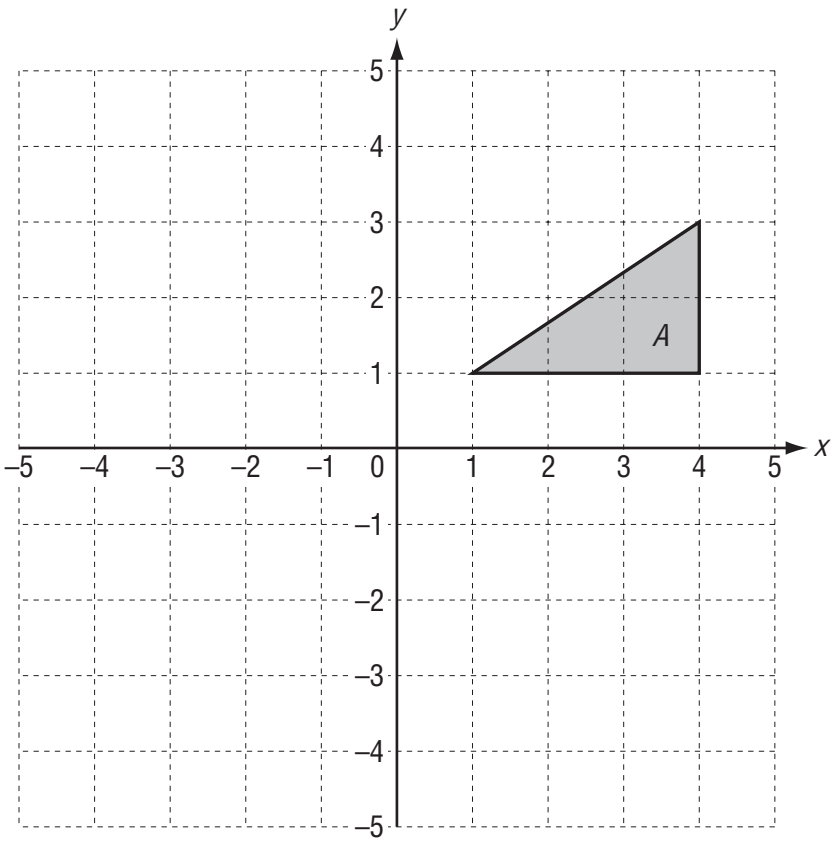
The triangle *KLM* is shown on the grid.

(a) Calculate angle *KML*.

Answer(a) Angle *KML* = ... [2]

(b) On the grid, draw the shear of triangle *KLM*, with a shear factor of 3 and the *x*-axis invariant. [2]

2 (a)



(i) Draw the image when triangle A is reflected in the line $y = 0$.
Label the image B. [2]

(ii) Draw the image when triangle A is rotated through 90° anticlockwise about the origin.
Label the image C. [2]

(iii) Describe fully the **single** transformation which maps triangle B onto triangle C.

Answer(a)(iii) ..... [2]

(b) Rotation through 90° anticlockwise about the origin is represented by the matrix $\mathbf{M} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(i) Find $\mathbf{M}^{-1}$, the inverse of matrix $\mathbf{M}$.

Answer(b)(i) $\mathbf{M}^{-1} = \begin{pmatrix} & \\ & \end{pmatrix}$ [2]

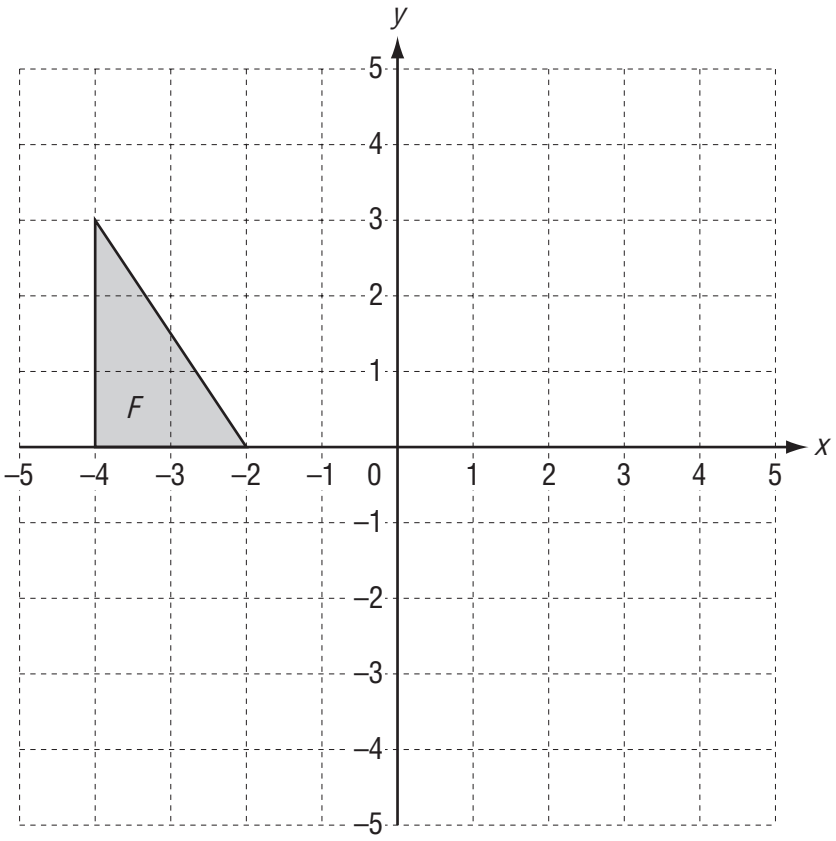
(ii) Describe fully the **single** transformation represented by the matrix $\mathbf{M}^{-1}$.

Answer(b)(ii) ..... [2]

(b) The area of triangle E is $k \times$ area of triangle A.
Write down the value of k.

Answer(b) $k =$ ..... [1]

(c)



(i) Draw the image of triangle F under the transformation represented by the matrix $\mathbf{M} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$. [3]

(ii) Describe fully this single transformation.

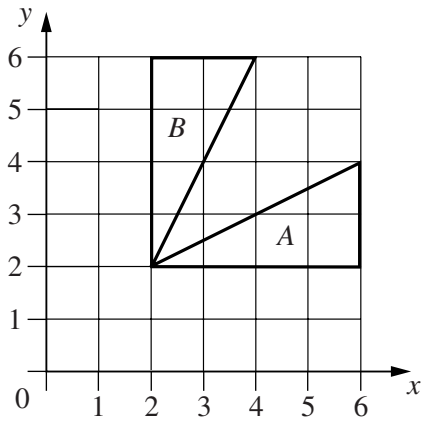
Answer(c)(ii) .....

..... [3]

(iii) Find $\mathbf{M}^{-1}$, the inverse of the matrix $\mathbf{M}$.

Answer(c)(iii) $\begin{pmatrix} & \\ & \end{pmatrix}$ [2]

19



(a) Describe fully the single transformation which maps triangle A onto triangle B.

Answer (a) ..... [2]

(b) Find the 2×2 matrix which represents this transformation.

Answer (b) [2]