## Trigonometry \& Bearings 2002-2011


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North


NOT TO
SCALE
$A O C$ is a diameter of the circle, centre $O$.
$A T$ is a straight line that cuts the circle at $B$.
$P T$ is the tangent to the circle at $C$.
Angle $C O B=76^{\circ}$.
(a) Calculate angle $A T C$.
(b) $T$ is due north of $C$.

Calculate the bearing of $B$ from $C$.


The diagram shows 3 ships $A, B$ and $C$ at sea.
$A B=5 \mathrm{~km}, B C=4.5 \mathrm{~km}$ and $A C=2.7 \mathrm{~km}$.
(a) Calculate angle $A C B$.

Show all your working.

Answer(a) Angle $A C B=$
(b) The bearing of $A$ from $C$ is $220^{\circ}$.

Calculate the bearing of $B$ from $C$.

6


The quadrilateral $A B C D$ represents an area of land.
There is a straight road from $A$ to $C$.
$A B=79 \mathrm{~m}, A D=120 \mathrm{~m}$ and $C D=95 \mathrm{~m}$.
Angle $B C A=26^{\circ}$ and angle $C D A=77^{\circ}$.
(a) Show that the length of the road, $A C$, is 135 m correct to the nearest metre.

Answer(a)
(b) Calculate the size of the obtuse angle $A B C$.
(c) A straight path is to be built from $B$ to the nearest point on the road $A C$.

Calculate the length of this path.

## Answer(c)

m [3]
(d) Houses are to be built on the land in triangle $A C D$. Each house needs at least $180 \mathrm{~m}^{2}$ of land.

Calculate the maximum number of houses which can be built. Show all of your working.

8


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SCALE

Parvatti has a piece of canvas $A B C D$ in the shape of an irregular quadrilateral.
$A B=3 \mathrm{~m}, A C=5 \mathrm{~m}$ and angle $B A C=45^{\circ}$.
(a) (i) Calculate the length of $B C$ and show that it rounds to 3.58 m , correct to 2 decimal places

You must show all your working.
Answer(a)(i)
(ii) Calculate angle $B C A$.
(b) $A C=C D$ and angle $C D A=52^{\circ}$.
(i) Find angle $D C A$.

$$
\begin{equation*}
\text { Answer(b)(i) Angle } D C A= \tag{1}
\end{equation*}
$$

(ii) Calculate the area of the canvas.
$\qquad$
$\mathrm{m}^{2}$
(c) Parvatti uses the canvas to give some shade.

She attaches corners $A$ and $D$ to the top of vertical poles, $A P$ and $D Q$, each of height 2 m . Corners $B$ and $C$ are pegged to the horizontal ground.
$A B$ is a straight line and angle $B P A=90^{\circ}$.


Calculate angle $P A B$.


The triangular area $A B C$ is part of Henri's garden.
$A B=9 \mathrm{~m}, B C=6 \mathrm{~m}$ and angle $A B C=95^{\circ}$.
Henri puts a fence along $A C$ and plants vegetables in the triangular area $A B C$.
Calculate
(a) the length of the fence $A C$,

$$
\begin{equation*}
\text { Answer }(a) A C= \tag{3}
\end{equation*}
$$

m
(b) the area for vegetables.

18


Antwerp is 78 km due South of Rotterdam and 83 km due East of Bruges, as shown in the diagram.

Calculate
(a) the distance between Bruges and Rotterdam,
(b) the bearing of Rotterdam from Bruges, correct to the nearest degree.

20 A plane flies from Auckland $(A)$ to Gisborne $(G)$ on a bearing of $115^{\circ}$. The plane then flies on to Wellington $(W)$. Angle $A G W=63^{\circ}$.

(a) Calculate the bearing of Wellington from Gisborne.
Answer (a)
(b) The distance from Wellington to Gisborne is 400 kilometres.

The distance from Auckland to Wellington is 410 kilometres.

Calculate the bearing of Wellington from Auckland.


The diagram shows three straight horizontal roads in a town, connecting points $P, A$ and $B$. $P B=250 \mathrm{~m}$, angle $A P B=23^{\circ}$ and angle $B A P=126^{\circ}$.
(a) Calculate the length of the road $A B$.
(b) The bearing of $A$ from $P$ is $303^{\circ}$.

Find the bearing of
(i) $B$ from $P$,
(ii) $A$ from $B$

12 The diagram represents the ski lift in Queenstown New Zealand.


NOT TO
SCALE
(a) The length of the cable from the bottom, $B$, to the top, $T$, is 730 metres.

The angle of elevation of $T$ from $B$ is $37.1^{\circ}$.

Calculate the change in altitude, $h$ metres, from the bottom to the top.
(b) The lift travels along the cable at 3.65 metres per second.

Calculate how long it takes to travel from $B$ to $T$.
Give your answer in minutes and seconds.


The diagram shows three points $P, Q$ and $R$ on horizontal ground.
$P Q=50 \mathrm{~m}, P R=100 \mathrm{~m}$ and angle $P Q R=140^{\circ}$.
(a) Calculate angle $P R Q$.
(b) The bearing of $R$ from $Q$ is $100^{\circ}$.

Find the bearing of $P$ from $R$.

5

$A B C D$ is a quadrilateral and $B D$ is a diagonal.
$A B=26 \mathrm{~cm}, B D=24 \mathrm{~cm}$, angle $A B D=40^{\circ}$, angle $C B D=40^{\circ}$ and angle $C D B=30^{\circ}$.
(a) Calculate the area of triangle $A B D$.

$$
\text { Answer }(a) \text {.......................................... } \mathrm{cm}^{2}
$$

(b) Calculate the length of $A D$.

> Answer(b)
cm
[4]
(c) Calculate the length of $B C$.

## Answer (c)

cm
(d) Calculate the shortest distance from the point $C$ to the line $B D$.

5


The diagram shows some straight line distances between Auckland $(A)$, Hamilton $(H)$, Tauranga $(T)$ and Rotorua $(R)$.
$A T=180 \mathrm{~km}, A H=115 \mathrm{~km}$ and $H T=90 \mathrm{~km}$.
(a) Calculate angle $H A T$.

Show that this rounds to $25.0^{\circ}$, correct to 3 significant figures.
Answer (a)
(b) The bearing of $H$ from $A$ is $150^{\circ}$.

Find the bearing of
(i) $T$ from $A$,

> Answer(b)(i)
(ii) $A$ from $T$.
(c) Calculate how far $T$ is east of $A$.

Answer(c)

km

[3]
(d) Angle $T H R=30^{\circ}$ and angle $H R T=70^{\circ}$.

Calculate the distance $T R$.
(e) On a map the distance representing $H T$ is 4.5 cm .

The scale of the map is $1: n$.
Calculate the value of $n$.

$$
\begin{equation*}
\text { Answer(e) } n= \tag{2}
\end{equation*}
$$

2

1 In the right-angled triangle $A B C, \cos C=\frac{4}{5}$. Find angle $A$.


2


In the quadrilateral $A B C D, A B=3 \mathrm{~cm}, A D=11 \mathrm{~cm}$ and $D C=8 \mathrm{~cm}$.
The diagonal $A C=5 \mathrm{~cm}$ and angle $B A C=90^{\circ}$.
Calculate
(a) the length of $B C$,

$$
\operatorname{Answer}(a) B C=\quad . . . . . . . . . . . . . . . \mathrm{cm}
$$

(b) angle $A C D$,
(c) the area of the quadrilateral $A B C D$.

10


In triangle $A B C, A B=12 \mathrm{~cm}$, angle $C=90^{\circ}$ and angle $A=27^{\circ}$.
Calculate the length of $A C$.

$$
\text { Answer } A C=
$$

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In the rectangle $A B C D, A B=9 \mathrm{~cm}$ and $B D=12 \mathrm{~cm}$.
Calculate the length of the side $B C$.

12 (a) Write 16460000 in standard form.
Answer(a)
(b) Calculate $7.85 \div\left(2.366 \times 10^{2}\right)$, giving your answer in standard form.

10 (a)

$B$ is 120 m from $A$ on a bearing of $053^{\circ}$.
Calculate
(i) the distance $d$,
(ii) the bearing of $A$ from $B$.

Answer(a)(i) $d=$
(b)

Answer(a)(ii)


A vertical flagpole, $A F$, is 9 m high.
It is held in place by two straight wires $F G$ and $F H$.
$F G=20 \mathrm{~m}$ and $A H=24 \mathrm{~m}$.
$G, A$ and $H$ lie in a straight line on horizontal ground.
Calculate
(i) angle $F H A$,

$$
\text { Answer(b)(i) Angle } F H A=
$$

(ii) the distance $G A$.

8 Manuel rows his boat from $A$ to $B$, a distance of 3 kilometres. The scale diagram below shows his journey. 1 centimetre represents 0.5 kilometres.

(a) (i) Measure the bearing of $B$ from $A$.

Answer(a)(i)
(ii) The journey from $A$ to $B$ takes him 30 minutes.

Calculate his average speed in kilometres per hour.

> Answer(a)(ii)
km/h
(b) From $B$, Manuel rows 3.5 kilometres in a straight line, on a bearing of $145^{\circ}$, to a point $C$.

On the diagram, draw accurately this journey and label the point $C$.
(c) Manuel then rows from $C$ to $A$.
(i) Measure $C A$.

> Answer(c)(i) ...................................... cm [1]
(ii) Work out the actual distance from $C$ to $A$.

Answer(c)(ii)
km [1]
(iii) By measuring a suitable angle, find the bearing of $A$ from $C$.

Answer(c)(iii)
(d) Two buoys, $P$ and $Q$, are on opposite sides of the line $A B$.

Each buoy is 2 km from $A$ and 1.5 km from $B$.
(i) On the diagram, construct and mark the positions of $P$ and $Q$.
(ii) Measure the distance between $P$ and $Q$.
Answer(d)(ii) ...................................... cm [1]
(iii) Find the actual distance, $P Q$, in kilometres.

4 (a)


The diagram shows triangle $F G H$, with $F G=14 \mathrm{~cm}, G H=12 \mathrm{~cm}$ and $F H=6 \mathrm{~cm}$.
(i) Calculate the size of angle $H F G$.
(ii) Calculate the area of triangle $F G H$.
(b)


The diagram shows triangle $P Q R$, with $R P=12 \mathrm{~cm}, R Q=18 \mathrm{~cm}$ and angle $R P Q=117^{\circ}$.
Calculate the size of angle $R Q P$.

3 (a)


The scale drawing shows the positions of two towns $A$ and $C$ on a map.
On the map, 1 centimetre represents 20 kilometres.
(i) Find the distance in kilometres from town $A$ to town $C$.

> Answer(a)(i)
$\qquad$
(ii) Measure and write down the bearing of town $C$ from town $A$.

## Answer(a)(ii)

(iii) Town $B$ is 140 km from town $C$ on a bearing of $150^{\circ}$.

Mark accurately the position of town $B$ on the scale drawing.
(iv) Find the bearing of town $C$ from town $B$.
Answer(a)(iv)
(v) A lake on the map has an area of $0.15 \mathrm{~cm}^{2}$.

Work out the actual area of the lake.

$$
\begin{equation*}
\text { Answer(a)(v) ............................................. } \mathrm{km}^{2} \tag{2}
\end{equation*}
$$

(b) A plane leaves town $C$ at 1157 and flies 1500 km to another town, landing at 1412 .

Calculate the average speed of the plane.

## Answer(b)

$\qquad$ km/h
[3]
(c)


The diagram shows the distances between three towns $P, Q$ and $R$.
Calculate angle $P Q R$.

4


NOT TO SCALE

The circle, centre $O$, passes through the points $A, B$ and $C$.
In the triangle $A B C, A B=8 \mathrm{~cm}, B C=9 \mathrm{~cm}$ and $C A=6 \mathrm{~cm}$.
(a) Calculate angle $B A C$ and show that it rounds to $78.6^{\circ}$, correct to 1 decimal place.

Answer(a)
(b) $M$ is the midpoint of $B C$.
(i) Find angle $B O M$.
(ii) Calculate the radius of the circle and show that it rounds to 4.59 cm , correct to 3 significant figures.

## Answer(b)(ii)

(c) Calculate the area of the triangle $A B C$ as a percentage of the area of the circle.


In the circle, centre $O$, the chords $K L$ and $P Q$ are each of length 8 cm . $M$ is the mid-point of $K L$ and $R$ is the mid-point of $P Q . \quad O M=3 \mathrm{~cm}$.
(a) Calculate the length of $O K$.
$\qquad$ .cm
(b) $R M$ has a length of 5.5 cm . Calculate angle $R O M$.


Felipe $(F)$ stands 17 metres from a bridge $(B)$ and 32 metres from a tree $(T)$.
The points $F, B$ and $T$ are on level ground and angle $B F T=40^{\circ}$.
(a) Calculate
(i) the distance $B T$,
(ii) the angle $B T F$.
(b) The bearing of $B$ from $F$ is $085^{\circ}$. Find the bearing of
(i) $T$ from $F$,
(ii) $F$ from $T$,
(iii) $B$ from $T$.
(c) The top of the tree is 30 metres vertically above $T$.

Calculate the angle of elevation of the top of the tree from $F$.


To avoid an island, a ship travels 40 kilometres from $A$ to $B$ and then 60 kilometres from $B$ to $C$.
The bearing of $B$ from $A$ is $080^{\circ}$ and angle $A B C$ is $115^{\circ}$.
(a) The ship leaves $A$ at 1155 .

It travels at an average speed of $35 \mathrm{~km} / \mathrm{h}$.
Calculate, to the nearest minute, the time it arrives at $C$.
(b) Find the bearing of
(i) $A$ from $B$,
(ii) $C$ from $B$.
(c) Calculate the straight line distance $A C$.
(d) Calculate angle $B A C$.
(e) Calculate how far $C$ is east of $A$.

$A, B$ and $C$ are three places in a desert. Tom leaves $A$ at 0640 and takes 30 minutes to walk directly to $B$, a distance of 3 kilometres. He then takes an hour to walk directly from $B$ to $C$, also a distance of 3 kilometres.
(a) At what time did Tom arrive at $C$ ?
Answer (a)
(b) Calculate his average speed for the whole journey.

Answer (b) $\qquad$ $\mathrm{km} / \mathrm{h}$
(c) The bearing of $C$ from $A$ is $085^{\circ}$.

Find the bearing of $A$ from $C$.
Answer (c)

6


The diagram shows the positions of London $(L)$, Dubai $(D)$ and Colombo $(C)$.
(a) (i) Show that $L C$ is 8710 km correct to the nearest kilometre. Answer(a)(i)
(ii) Calculate the angle CLD.

2


The diagram shows five straight roads.
$P Q=4.5 \mathrm{~km}, Q R=4 \mathrm{~km}$ and $P R=7 \mathrm{~km}$.
Angle $R P S=40^{\circ}$ and angle $P S R=85^{\circ}$.
(a) Calculate angle $P Q R$ and show that it rounds to $110.7^{\circ}$.

Answer(a)
(b) Calculate the length of the road $R S$ and show that it rounds to 4.52 km .

Answer(b)
(c) Calculate the area of the quadrilateral $P Q R S$.
[Use the value of $110.7^{\circ}$ for angle $P Q R$ and the value of 4.52 km for $R S$.]


Theresa swims from $P$ to $Q$, then from $Q$ to $R$ and then finally returns from $R$ to $P$. $P Q=140 \mathrm{~m}, R P=220 \mathrm{~m}$ and angle $P R Q=31^{\circ}$.
(a) Angle $P Q R$ is obtuse.

Calculate its size, to the nearest degree.

> Answer (a)
(b) The bearing of $Q$ from $P$ is $060^{\circ}$. Calculate the bearing of $R$ from $Q$.
Answer (b)

22 f: $x \mapsto 3-2 x \quad$ and $\quad \mathrm{g}: x \mapsto \frac{x+1}{4}, \quad$ for all values of $x$.
(a) Find $\mathrm{f}\left(-\frac{3}{4}\right)$.

> Answer (a)
(b) Find the inverse function, $\mathrm{g}^{-1}(x)$.

$$
\text { Answer }(b) \mathrm{g}^{-1}(x)=
$$

(c) Find the composite function, $\mathrm{fg}(x)$, giving your answer as a single fraction.

